

AN ABSTRACT OF THE THESIS OF

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This thesis presents a discussion of motion of a six-legged walking machine following removal of one leg constraint. To take a step, one leg must be lifted and placed at some other position. As soon as the constraint provided by the leg is removed, however the machine begins to fall. This falling motion can be represented as screw motion of the body center of mass and of body-leg attachments.

First a study of body workspace of the machine was done with all six feet on the ground. Body workspace is the intersection of kinematic and force workspaces. Kinematic workspace is the volume in space where the center of mass can be placed such that all joint angles of legs are within specified limits; and force workspace is the volume in space where the force

in all legs is compressive. The affect of various foot positions, pitches and heights of the center of mass were investigated to find a set of three symmetric foot positions that might constitute acceptable intermediate positions in a walking sequence.

Motion of the center of mass in the forward direction is limited in the force workspace by two points at which the force on a pair of legs goes to zero. With the center of mass at each of these two positions, the screw parameters resulting from the release of force on one leg (front, middle and hind individually) could be determined.

Dynamic simulation of these body and foot positions used the commercial software SD/FAST. Code was written in C to do both static and dynamic simulation of machine and merged with code generated by SD/FAST. Code was also written in AutoLisp to plot the falling motion of machine.

Screw parameters found in this study were such that sustained forward motion of the body could not be achieved using the falling motion alone. Other measures such as extending one or more of the remaining five legs would be needed for effective forward body motion.

Initial Spatial Motion
of a Rigid Body
on Removal of One Constraint

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Initial Spatial Motion of a Rigid Body on Removal of One Constraint

1. Introduction

1.1 Objective

The study deals with investigating the motion of a rigid body when one constraint on the body is removed. The six legged walking machine being built at OSU was considered as the object of investigation. The walking machine has six degrees of freedom, one degree of freedom coming from each leg. In order to make the machine move, each leg must be lifted and placed, in turn, at some other suitable position. If five legs of the machine are restrained from moving, and one leg is lifted in the process of taking a step, the machine has one degree of freedom. Since there are number of forces acting on the machine, the machine tends to fall and moves from one position to another. This motion of machine can be described as a screw motion, that is, rotation about a screw axis and translation along a screw axis. The study deals with finding the screw parameters, for the motion of the machine during step initiation. The screw parameters include; screw axis, magnitude of rotation about screw axis and

magnitude of translation along screw axis. The study also compares the screw parameters for different sets of machine parameters.

Before the machine is in a position to lift one leg and move, it is important to find the body workspace of the walking machine. Body workspace of machine is intersection of kinematic workspace and force workspace. Body workspace of machine thus gives the positions where the machine can practically stand. Body workspace of machine gives suitable positions where one leg can be lifted to initiate a step.

Thus the study can be divided into two main objectives, which are :

- (i) Determination of Body Workspace of six-legged walking machine.
- (ii) Calculation of instantaneous screw axis by doing dynamic simulation of the machine using SD/FAST.

1.2 Why Walking Machines ?

Over the last two decades, a lot of research has been done on walking machines. There are many reasons for studying walking machines. One of the main reasons is mobility, that is, there is need for machines that can travel in difficult terrains, where existing vehicles cannot go. Though

machines with wheels can travel on even surfaces, it is difficult for them to move on uneven surfaces or rough terrain. In rough terrain, legs provide much better mobility than wheels. Walking machines, like walking animals will be capable of travelling in difficult terrains such as; climbing steep inclines, traversing narrow beams and manoeuvring around obstacles. Some of the many uses of walking machines are the maintenance of space stations, nuclear stations and underwater structures.

In addition, walking machines also have advantage of having greater efficiency in rough terrain as compared to wheeled vehicles. Legs of walking machines may also be used as secondary arms for gripping or digging. Legs can also perform an important function as counter weight to restore an overturned body to its feet (Todd 1985). A legged vehicle can achieve a smooth motion on rough ground by varying the effective lengths of its legs to match the unevenness of the ground. Also legs do less damage to the ground than wheels.

Another main reason for studying walking machines is to understand human and animal locomotion. Both humans and animals demonstrate great mobility and agility. Humans can carry, swing, toss and propel their bodies through space, maintaining balance, orientation and speed. Animals also can move quickly and rapidly through forests, swamp, jungles etc.

1.3 Why Darkling Beetle Was Chosen as a Model?

Vertebrates and arthropods have multi-segmented, articulated legs. Arthropods, including insects, spiders and crabs, have simple nervous system and an external skeleton system, that is the joints are on the outside, not covered by layers of skins and muscles, thus making it easier to study and locate the joints. Whereas vertebrates have much more sophisticated nervous system and an internal skeleton, that is the joints are covered by layers of skin and muscles.

Over the last 300 million years arthropods have adapted to deal with wide variety of terrain (Fichter et al. 1987). Because of their small size and the rigorous competition involved in survival, arthropods have become efficient machines. Most arthropod leg joints are revolute joints made up of pairs of ball-and-socket joints. External structure, rigid body and leg segments connected by simple joints, is very similar to a walking machine.

An ideal model for study, for building a walking machine, would be an arthropod whose locomotion is confined to walking or running. Ease of handling and inability to jump or fly are the other two main criteria for selecting an animal as a model for designing a walking machine. It is also desirable for a walking machine to possess good balance when at rest and when walking.

The darkling beetle, *Eleodes Obscura Sulcipennis*, was chosen as the model for walking machine design. This insect is unable to fly or jump, is about 30 mm long from head to tail, is long lived and easy to handle and analyze. Darkling beetles are easily available and commonly found in arid regions of the western USA.

1.4 Why Study Body Workspace ?

Kinematic workspace is defined as the range of body motion with constraints on the joint angles at the legs, and feet fixed to ground. Force workspace is defined as the range of body motion when forces in all legs act towards the body, that is forces in all legs counter balance the weight of the body. For determining force workspace it was assumed that legs apply only force and no moment to the body, that is connection of leg to body transmits no moment. Given foot positions and kinematic parameters of walking machine, position of the mass center of machine can be mapped to find both the kinematic workspace and the force workspace. The intersection of kinematic workspace and force workspace gives the body workspace. Thus body workspace is the volume in space where mass center of the body can be placed such that joint angles of the legs are within their ranges and also the forces in all the six legs are compressive, that is, the forces in all the legs counter balance the body-weight.

It is important to study the body workspace so as to find the best combination of parameters such as foot positions, body pitch and body height, to make the machine stand. The larger the body workspace for a particular set of parameters the better is the combination of the parameters. Larger body workspace gives larger area in which the body can be made to stand. It also gives a good idea of area where the body can move. Body workspace also gives the positions where the center of mass of machine can be placed such that the force in one of the leg is zero.

1.5 Why Study Falling Screw ?

After having found body workspace the next step is to study the direction of fall when one leg is lifted to initiate a step. Body workspace gives the position where the center of mass of machine can be placed such that force in one of the leg is nearly zero, and then that leg of the machine can be lifted to initiate a step. As one leg is lifted the body of the machine tends to fall, under the influence of forces exerted by legs and its own weight, and thus moves from one position to the other. This motion of machine can be represented as a screw motion. Thus to completely define this falling motion of machine the screw parameters need to be found. Screw parameters include; screw axis, magnitude of rotation about the screw axis and magnitude of translation along the screw axis.

1.6 Organization of Study

This thesis is divided into six parts. The first part, presented in chapter 2, gives the background of the research done on the walking machine. It also discusses the kinematic parameters of the beetle and the machine.

The model developed for use in SD/FAST, the software used for simulation, is discussed in chapter 3. The chapter explains in details the conversion of the model of machine from A-model to the model accepted by SD/FAST. It also explains the capabilities of SD/FAST.

Chapter 4 discusses the body workspace of the machine with variation in foot positions, body height and body pitch. It also explains the changes in the design parameters done to get better results in the body workspace.

Chapter 5 discusses the screw axis determination for the combination of parameters, for which the body workspace was found to be relatively large. The screw axis is found at the end of workspace where the force in one leg is nearly zero.

Chapter 6 presents recommendations for possible future subjects for investigation. It also discusses changes in design which can be implemented to get better results in terms of body workspace and screw parameters.

2. Kinematic Parameters of Beetle and Machine

The design of walking machine is based on beetle. A thorough study of beetle has been done (Fichter et al 1988, Albright et al 1988). The bottom and side view of the beetle are shown in Figure 2.1 and Figure 2.2 respectively. In Figure 2.1 B_{xy} is the body coordinate system of the beetle; L1 and R1 are the left front and right front legs; L2 and R2 are the left middle and right middle legs; L3 and R3 are the left hind and right hind legs. Figure 2.1 also shows the leg segments of the beetle. In Figure 2.2 B_{yz} is the body coordinate system and G_{yz} is the ground coordinate system.

Each leg of beetle is composed of five segments; coxa, trochanter, femur, tibia and tarsus. The first four segments, that is, coxa, trochanter, femur and tibia are joined together by revolute joints while the tibia and tarsus are joined by a ball-and-socket joint. It was found that trochanter has very limited motion relative to femur, and thus it is assumed that the trochanter is fused with femur. Thus the joint between coxa and trochanter is referred to as the joint between coxa and femur. It was also found that tarsus and all of its subsegments contact ground during walking and these segments exert very small force on the body. Thus the tarsus was altogether neglected for studying both kinematic range of motion and force

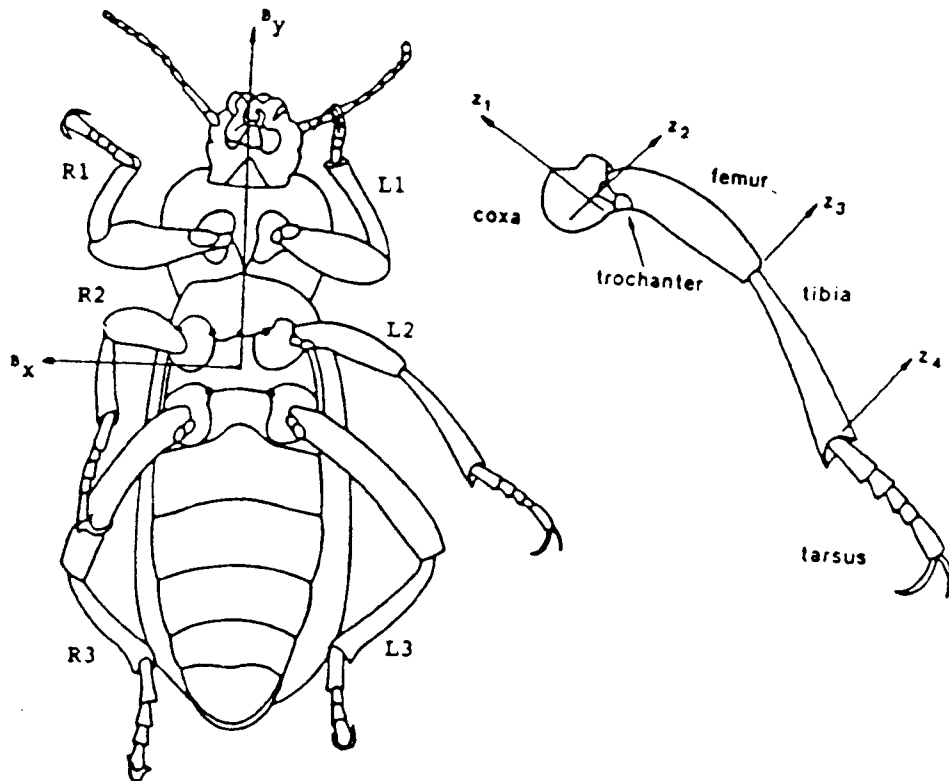


Figure 2.1 : Ventral view of a darkling beetle (left), and close-up of the left middle leg showing segment names and joint axes.

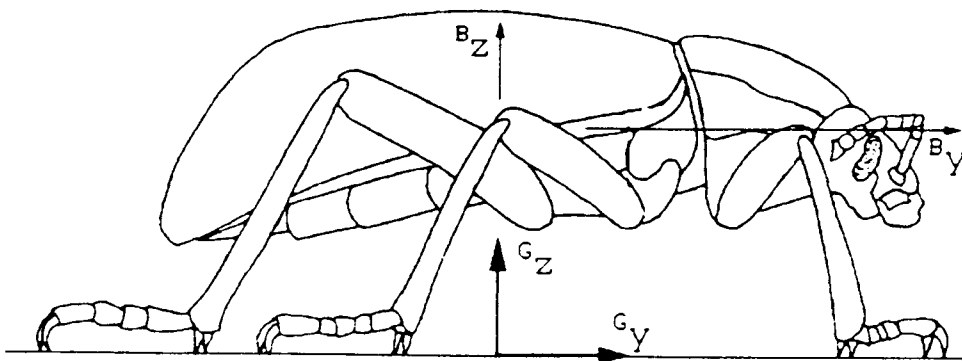


Figure 2.2 : Side view of a darkling beetle showing global and body coordinate systems.

workspace. For the study of body workspace a point contact between tibia and ground was assumed and for modeling point of view the joint was assumed to be a passive ball-and-socket joint. Thus the leg of beetle is reduced to three revolute joints, which are body-coxa, coxa-femur and femur-tibia. This configuration is similar to a RRR manipulator. In Figure 2.1 z_1 , z_2 and z_3 are the joint axes of joints body-coxa, coxa-femur and femur-tibia respectively.

For designing the machine it was very important to measure the kinematic parameters of beetle as accurately as possible. Determination of kinematic parameters of the beetle was done by following the procedure explained in Fichter et al, 1988. The kinematic parameters of the beetle were determined in both the D-H parameter (Craig, 1988) and A-model (Albright et al, 1991).

2.1 A-Model

A-model (Albright et al, 1988) was developed, to satisfy three main criteria, which are proportionality, similarity and expendability. This model is used here for describing the kinematic parameters of walking machine and beetle.

It had been observed (Albright et al, 1991) that, while using D-H parameters, a small difference in the parameters results in a large change in both the link offset and joint angle, especially when the joint axes are nearly parallel to each other. Thus with D-H parameters, variation in the model parameters does not proportionally reflect difference in links, that is, it does not satisfy the criterion of proportionality.

In D-H parameters the distance between two adjoining axes is taken as the common perpendicular or the shortest distance. As a result the distance does not give any idea of the actual shape and length of the links, thus not satisfying the criterion of similarity, which requires that the parameters directly represent physical properties of the links. A-model allows origins of coordinates frames to be placed any where along the joint axis; thus model parameters can be made comparable to physical dimensions of links.

It is also important for a model to be expandable, that is, it should be capable of representing both single degree of freedom and multiple degree of freedom joints. This is achieved by separating the description of joints and links. This also helps in separating the link constants and joint constants. In this thesis only revolute-revolute links are discussed as the joints of arthropods are very simple. Most of the arthropods have either single degree of freedom revolute joints or have ball-and-socket joints.

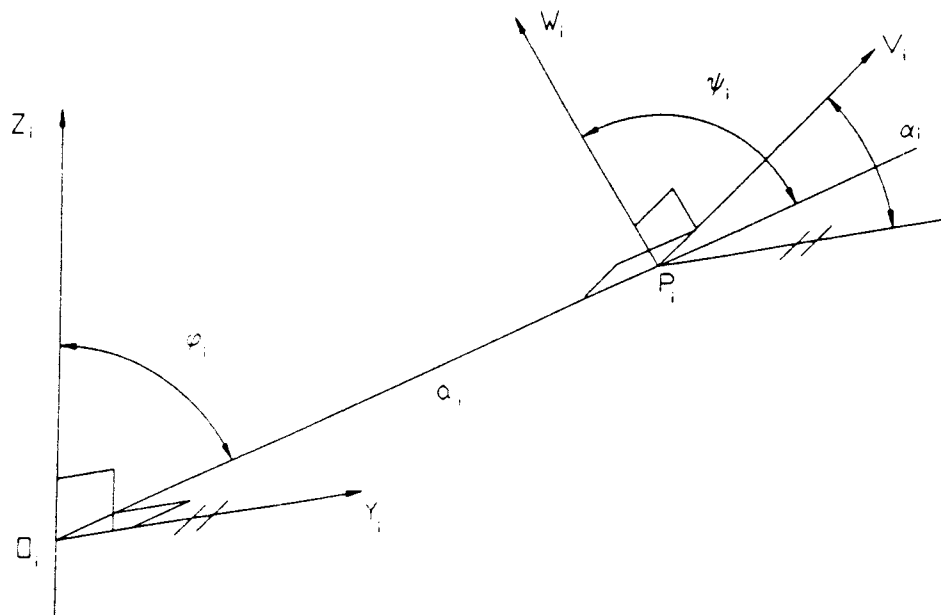


Figure 2.3 : A-Model parameters for i^{th} link.

A-model uses two coordinate frames in each of the segment, which are xyz_i called the proximal coordinate frame and uvw_i called the distal coordinate frame. As shown in Figure 2.3 the unit vector z_i passes through point o_i (origin of coordinate frame xyz_i) and is called the proximal joint axis. The unit vector w_i passes through point p_i (origin of coordinate frame uvw_i) and is called the distal joint axis. The vector from o_i to p_i is defined as a_i .

Axis y_i is perpendicular to axis z_i and unit vector along a_i , while axis v_i is perpendicular to axis w_i and unit vector along a_i . The angle between the vector a_i and z_i is defined as proximal angle (ϕ_i), and the angle between a_i and w_i is defined as distal angle (ψ_i). The values of ϕ_i and ψ_i range from 0 to π . The twist angle (α_i) is defined as the angle between y_i and v_i . It ranges from $-\pi$ to π . Figure 2.3 shows the above parameters in details (Albright et al 1990).

These parameters define the transformation from coordinate frame xyz_i to uvw_{i-1} . For a revolute joint, origins of frame uvw_{i-1} and xyz_i are coincident, axes w_{i-1} and z_i coincide and have the same sense, and joint variable Θ_i is the angle between unit vector u_{i-1} and unit vector x_i .

For a beetle, a measurement coordinate (xyz_m) frame was placed at the line connecting ventral articulation of the middle leg pair. Axis x_m lies along this line segment and is positive towards the right of the animal. Axis y_m intersects the line connecting the ventral articulation of the hind leg pair and is positive towards the head of the beetle. Axis z_m completes the right handed coordinate system.

On each leg segment axis z_i is the proximal joint axis, w_i is the distal joint axis. As explained above for a revolute pair, axes w_{i-1} and z_i coincide and have the same sense. Origin of xyz_i and uvw_{i-1} coincide and can be

Table 2.1 : Average kinematic parameters of right legs of five darkling beetle.

			Parameters		
			Front	Middle	Hind
Body-Coxa Joints (in measuring frame)					
Origin of coxa coordinate frame, uvw_0	x_m (mm)		0.62	1.61	2.25
	y_m (mm)		4.06	-0.28	-3.41
	z_m (mm)		0.95	0.90	1.09
w_0 -axis unit vector	l		-0.518	-0.458	-0.488
	m		-0.071	0.151	0.359
u_0 -axis unit vector	l		-0.128	0.300	0.582
Leg Segments					
Coxa	Joint range	(deg)	156.5	108.2	69.3
	Joint minimum	(deg)	-42.9	45.4	92.6
	Joint maximum	(deg)	121.6	153.6	161.9
	Segment length	(a_i) (mm)	0.42	1.46	1.73
	Proximal angle	(ϕ_i) (deg)	90.0	90.0	90.0
	Distal angle	(ψ_i) (deg)	90.0	90.0	90.0
	Twist angle	(α_i) (deg)	-98.5	91.7	94.5
Femur	Joint range	(deg)	138.8	124.4	132.0
	Joint minimum	(deg)	-36.1	-86.4	-83.5
	Joint maximum	(deg)	94.7	38.4	47.5
	Segment length	(a_i) (mm)	7.65	8.32	11.34
	Proximal angle	(ϕ_i) (deg)	58.1	81.8	75.0
	Distal angle	(ψ_i) (deg)	67.7	85.5	75.7
	Twist angle	(α_i) (deg)	20.3	0.7	-0.4
Tibia	Joint range	(deg)	144.9	145.3	149.8
	Joint minimum	(deg)	-173.7	12.5	9.5
	Joint maximum	(deg)	-25.1	158.4	159.7
	Segment length	(a_i) (mm)	6.59	8.73	10.42
	Proximal angle	(ϕ_i) (deg)	83.9	91.6	89.0

Table 2.2 : Kinematic parameters of right legs of machine
(approximately 40 times the size of beetle).

		Parameters		
		Front	Middle	Hind
Body-Coxa Joints (in body frame)				
Origin of coxa coordinate frame, uvw_0	x_m (mm)	50.00	90.00	115.00
	y_m (mm)	235.00	65.00	-60.00
	z_m (mm)	-100.00	-100.00	-100.00
w_0 -axis unit vector	l	-0.500	-0.433	-0.500
	m	0.000	0.250	0.500
	n	-0.866	-0.866	-0.707
u_0 -axis unit vector	l	0.000	0.500	0.707
	m	1.000	0.866	0.707
Leg Segments				
Coxa	Joint range (deg)	160.00	160.00	70.00
	Joint minimum (deg)	-40.00	40.00	90.00
	Joint maximum (deg)	120.00	160.00	160.00
	Segment length (a_i) (mm)	0.00	0.00	0.00
	Proximal angle (ϕ_i) (deg)	90.00	90.00	90.00
	Distal angle (ψ_i) (deg)	90.00	90.00	90.00
	Twist angle (α_i) (deg)	-90.00	90.00	90.00
Femur	Joint range (deg)	120.00	120.00	120.00
	Joint minimum (deg)	-30.00	-90.00	-90.00
	Joint maximum (deg)	90.00	30.00	30.00
	Segment length (a_i) (mm)	300.00	350.00	450.00
	Proximal angle (ϕ_i) (deg)	90.00	90.00	90.00
	Distal angle (ψ_i) (deg)	90.00	90.00	90.00
	Twist angle (α_i) (deg)	0.00	0.00	0.00
Tibia	Joint range (deg)	140.00	140.00	140.00
	Joint minimum (deg)	-160.00	20.00	20.00
	Joint maximum (deg)	-20.00	160.00	160.00
	Segment length (a_i) (mm)	250.00	350.00	400.00
	Proximal angle (ϕ_i) (deg)	90.00	90.00	90.00
	Distal angle (ψ_i) (deg)	90.00	90.00	90.00
	Twist angle (α_i) (deg)	0.00	0.00	0.00

placed at an arbitrary point. Body-coxa and coxa-femur joints are nearly perpendicular, making it convenient to place the coordinate frame origins at the intersection of these joint axes with their common perpendicular. Since both articulations of femur-tibia joints can be seen, uvw_2 and xyz_3 coordinate frames origins are positioned midway between them. The origin of frame uvw_3 is placed at the center of tibia-tarsus joint, a ball-and-socket joint. The average kinematic parameters of right legs of five darkling beetles are listed in Table 2.1.

These parameters were then simplified and made forty time the size to get the kinematic parameters for the machine. Kinematic parameters for the machine are listed in Table 2.2. The parameters were simplified by making the coxa length equal to zero, thus making the joints body-coxa and coxa-femur equivalent to a Hooke joint with 90 degree twist and the femur twist zero. The position of the hind leg Hooke joint is also moved further back on the body. This has been done to give foot larger range of motion while keeping legs in compression and maintaining the wrench support model (Fichter et al, 1988). Since feet are only point contact they cannot support tension on the legs. For the machine maximum and minimum limits of joints angles were also changed a little, and to a certain extend depended on the physical dimensions of legs.

3. SD/FAST Software

SD/FAST is commercially available software developed by Symbolic Dynamics, Inc. It can be used to perform analysis and design studies on any mechanical system, for example multiple arm robot, walking machines, mechanisms and machines etc. Broadly any mechanical system which can be modeled as a set of rigid bodies interconnected by joints can be analyzed using SD/FAST. The systems can be driven by prescribed motion (constant speed motors, sinusoidal motion, locked joints etc.), influenced by forces (gravity, hydraulic forces, friction etc.) or restricted by constraints (complex cams, screw joints etc.). Most of the analysis can be done with SD/FAST and a small amount of user-written code in C or FORTRAN. The assembly analysis, velocity analysis, forward and inverse dynamics, static force analysis etc. can be easily done with SD/FAST.

For all kinds of model development the only type of units assumed by SD/FAST is for angular displacement, which is radian. All other units including mass, length and time units are left unspecified. It is user's responsibility to use consistent units. For example, if the units of mass is kg, unit of length is meter and unit of time is second, the unit of force will be kg.m/s^2 .

3.1 Pendulum

The first step in using SD/FAST is to make a model or system description file. The main steps in developing the system description file are explained below with an example of defining a pendulum.

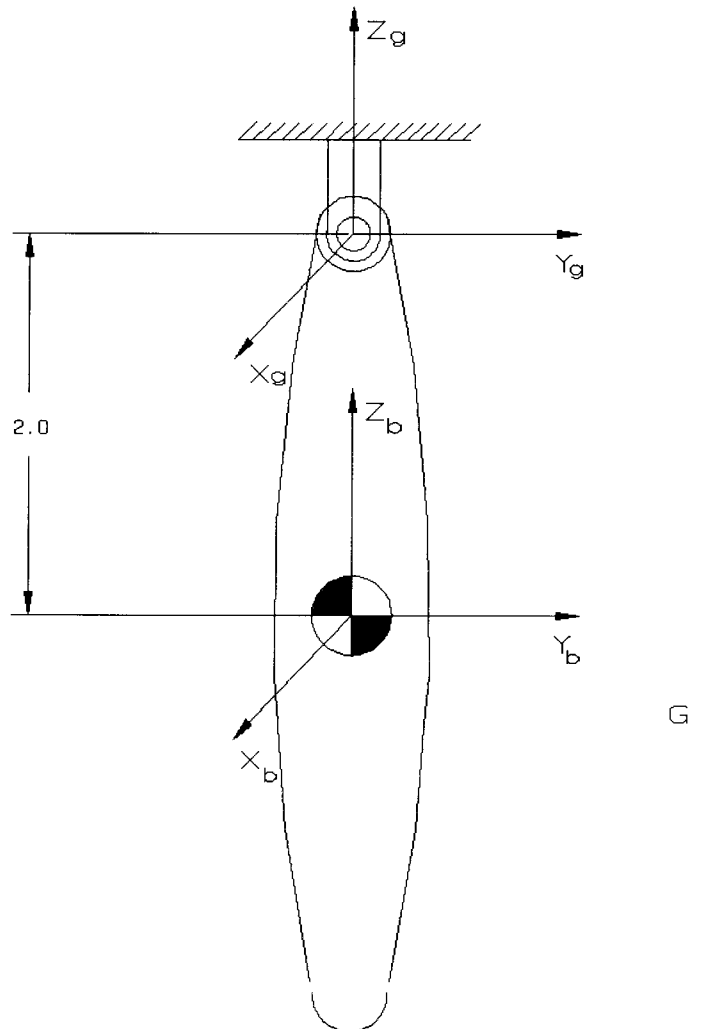


Figure 3.1 : Drawing of pendulum, for developing system description file for SD/FAST.

(i) Draw picture and define topology : Figure 3.1 shows a simple pendulum, one body in an open-tree structure, connected to ground by a pin joint. There is gravity acting on the whole system.

(ii) Define coordinate frames : In this step all the coordinate systems are defined. The body coordinate system is placed at the center of mass of the body and the ground coordinate system can be placed at any convenient position. For the pendulum the ground coordinate system is placed at the pin joint. In Figure 3.1 xyz_g is the ground coordinate system and xyz_b is the body coordinate system. The vector \mathbf{G} gives the direction of gravity acting on the whole system.

(iii) Specify geometry and joints : After specifying the coordinate frames, geometry and joint specifications are required. The geometry and joint information is given in terms of vectors. For any system the only information needed to specify the geometry is the vector from the mass center to the joint. For the pendulum shown in Figure 3.1 the vector from mass center to the joint is :

$$\mathbf{r} = 0 \mathbf{e}_1 + 0 \mathbf{e}_2 + 2.0 \mathbf{e}_3$$

The orientation of the joint axis is given by :

$$\mathbf{a} = 1.0 \mathbf{e}_1 + 0 \mathbf{e}_2 + 0 \mathbf{e}_3$$

The joint axis is always a unit vector. User can specify a non-unit vector, but SD/FAST converts it into a unit vector while developing the code for analysis.

(iv) Specify mass properties : Each body in the system must have a mass and inertia matrix specified. For the pendulum mass of 10 kg is chosen. The inertia matrix must be specified about the mass center of the body in the body coordinate frame. For the pendulum the inertia matrix chosen is

$$\mathbf{I} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ kg.m}^2$$

(v) Specify constraints and prescribed motion : The motion constraint imposed by the pin joint has already been specified. For this example no other constraint is needed.

(vi) Specify gravity : The constant known force is specified in the input file so that the symbol manipulator can incorporate the known forces in the equations of motion. Also the forces that are acting on all the bodies of system, for e.g. gravity, should be specified in the input file.

After having done the above six steps the next step is to write system description file. It contains all the information discussed above. The system description file is free format, and the details of the format are discussed in the manual for the software (SD/FAST, 1991). For the pendulum the system description file is shown in Figure 3.2.

```
#
# Input file for pendulum
# Name of file : pend.sd

gravity = 0 0 -9.8          } Preamble

body      = pendulum        } start of body paragraph
inb       = $ground
joint     = pin
mass      = 10?
inertia   = 5 5 1
bodytojoint = 0 0 2
inbtojoint = 0 0 0
pin       = 1 0 0
```

Figure 3.2 : System description file for pendulum, developed for SD/FAST.

Any line beginning with '#' is treated as a comment statement and is not processed by SD/FAST. For the above figure right braces and the statements after right braces are not included in the system description file. The preamble contains any information that applies to the entire system. In this case only information on gravity is included. Other information as "single" for generating single precision code and language option can be placed in the preamble. The gravity vector is specified in the ground coordinate frame and is given by :

$$\mathbf{G} = 0 \mathbf{e}_1 + 0 \mathbf{e}_2 - 9.8 \mathbf{e}_3$$

After specifying all the information of preamble, body paragraph is written, which contains all the information of the body being described. The keyword "body" is followed by a name and is any name given to the body by the user. This is useful when describing more than one body in the file. All the information listed after "body = ", but before the next "body = " is assumed to apply to the specific body only. The next phrase "inb = \$ground" tells SD/FAST to which body this body is being connected. The body "\$ground" is built-in and contains the ground frame. A body must be defined before it can be used as an inboard body. The next phrase "joint = pin" tells SD/FAST about the type of joint between the body and its inboard body, which, in this case, is a pin or revolute joint. The vector from the center of mass of the body to the joint is specified by the phrase

"bodytojoint = ", and the vector from the inboard body frame to the joint is specified by the phrase "inbtojoint = ". For the example of pendulum the vectors are taken as explained above. "pin = " specifies the direction of axis of the joint. The phrases "mass = " and "inertia = " specifies the mass and inertia properties of the body being described. The "?" mark after the "mass = 10" phrase means that the mass of the body may be changed by the user while running the simulation. The "?" may appear after any number in the system description file, which will help user to change that number in the code. If there is no "?" in the system description file then the property cannot be changed in the code and the user will have to change the property in the system description file and run SD/FAST again to generate the system equations with changed parameters. In the system description file the "inertia = " contains only three numbers, in which case it is assumed by SD/FAST that the inertia specified is about the principal axes.

After having written the system description file the user is ready to run SD/FAST and generate the equations of motion for the system. When SD/FAST is run it creates four files which are either in FORTRAN or C depending on the user choice as indicated in the preamble of the system description file. The language option can also be specified at the command line, that is, when running SD/FAST. The default option is FORTRAN, that is, SD/FAST generates the code in FORTRAN if the language option is not specified by the user.

The names of the files created by SD/FAST depends on the name of the system description file (in this case pend.sd). The files created are :

(i) pend_i : This file contains general information of the model described in the system description file. This file also contains the information to find the reference numbers and array dimensions for making calls to SD/FAST generated routines. This file contains a Roadmap, which describes the system topology as a cross-check of the input System Description file, a State Index Map which maps the "position" states, the q's, and the "velocity" states, the u's, to their locations in the state vector, and a list of system parameters and their values. The information file is described in detail in the manual for the software (SD/FAST, 1991). The information file for the pendulum example is included in the Appendix.

(ii) pend_s.for or pend_s.c : This file contains the SD/FAST generated simplified analysis routines. The file is a code written in FORTRAN or C, as indicated by the extension, depending on the choice of the user. The routines contained in this file allow the user to perform most of the analysis tasks, but do not provide completely general analysis capabilities. This file contains the code for doing assembly analysis, motion analysis, static analysis etc.

(iii) `pend_d.for` or `pend_d.c` : This file contains the SD/FAST generated dynamics simulation code. This file contains all the system equations and system-specific generated code, written in FORTRAN or C. If the system description file is big, that is, the system of bodies being described is big, the code generated for dynamic simulation is broken down into six files. The files would then be named as `pend_d.c`, `pend_d00.c`, `pend_d01.c`, `pend_d02.c`, `pend_d03.c`, `pend_d04.c` and `pend_d05.c`. All these files have to be compiled and linked with other files to get an executable file.

(iv) `sdlib.for` or `sdlib.c` : This file generated by SD/FAST contains purely numerical, problem independent library routines. The routines contained in this file are used by the routines in the Simplified analysis file and the Dynamic simulation file. The library file generated is independent of the system modeled and thus need be generated only once. This file does not take the name of the system description file.

After having generated the files, user has to write code for doing the analysis required. For writing the code, user has to write at least one routine which is "sduforce" which contains the forces acting at any point on the system or torques acting at a joint in the system. The routines should be written and left blank if there are no forces and torques acting on the system. This is because SD/FAST can not determine from the input file if

the user wants to apply forces or torques to the system, but it can determine if prescribed motions or user constraints are needed. The details of writing the subroutine are explained in the manual of the software (SD/FAST, 1991).

3.2 Four-Bar Mechanism

This example explains system description file for a four bar mechanism, which consists of ground, a crank, a connecting rod and a rocker. As explained above, the six steps for developing the model, before writing the system description file, are followed. Figure 3.3 shows the four-bar mechanism model. In Figure 3.3 xyz_g is the ground coordinate system; xyz_c represent coordinate system for crank; xyz_r is the rocker coordinate system and xyz_{cr} is the coupler coordinate system. The coordinate systems are located in the middle of the length of the respective links. The system description file for the mechanism is shown in Figure 3.4.

As explained in the pendulum example the preamble contains the information which applies to the whole system. For the four bar mechanism the preamble contains the gravity vector acting on the system. It also contains the "language = c " option which means that the user wants to use C language for the code generation, that is, all the code generated by SD/FAST will be in C.

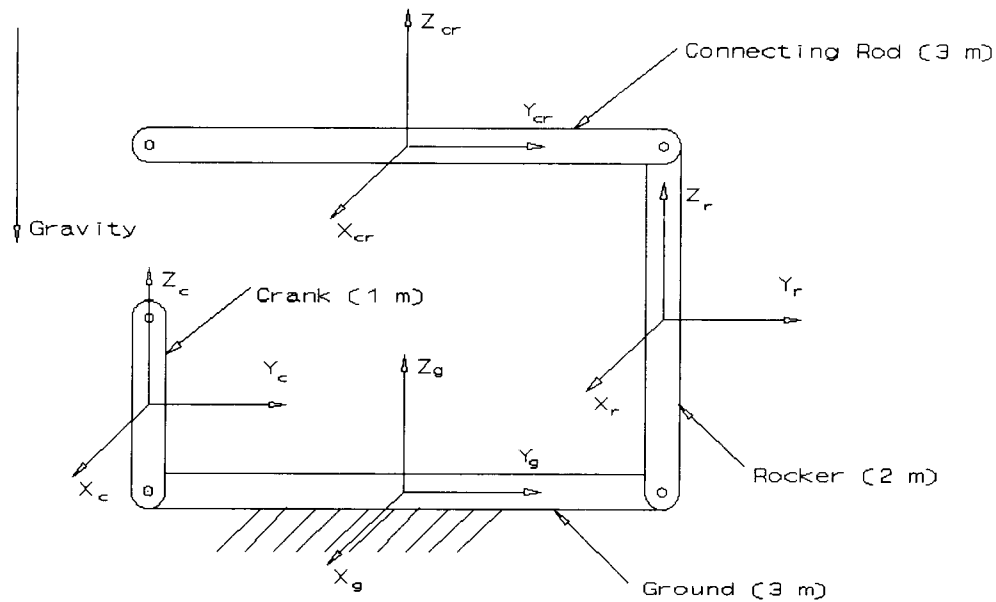


Figure 3.3 : Drawing of Four-Bar mechanism, for developing system description file for SD/FAST.

The first body defined is the crank ("body = crank"). Crank is connected to ground ("inb = \$ground") by a pin joint, and has a mass of 1 unit. Coordinate system for crank is located at its center of mass. Vector from center of mass of body being described (crank) to the joint is given by "bodytojoint = 0 0 -0.5". The ground coordinate system is located in the middle of link length, therefore vector from inboard body (\$ground) is given by "inbtojoint = 0 -1.5 0". The direction of the pin vector is given by "pin = 1 0 0". Since next statement in the system description file starts with keyword "body = ", the description for next body starts.


```

# Input file for four bar mechanism
# Name of file : fbar.sd

gravity = 0 0 -9.8
language = c

body = crank
inb = $ground
joint = pin
mass = 1
inertia = 1 0 1
bodytojoint = 0 0 -0.5
inbtojoint = 0 -1.5 0
pin = 1 0 0

body = rocker
inb = $ground
joint = pin
mass = 2
inertia = 2 0 2
bodytojoint = 0 0 -1
inbtojoint = 0 1.5 0
pin = 1 0 0

body = connect
inb = rocker
joint = pin
mass = 3
inertia = 0 3 3
bodytojoint = 0 1.5 0
inbtojoint = 0 0 1
pin = 1 0 0

# This is a loop joint

body = connect
inb = crank
joint = pin
bodytojoint = 0 -1.5 0
inbtojoint = 0 0 0.5
pin = 1 0 0

```

Figure 3.4 : System description file of four-bar mechanism, developed for SD/FAST.

Rocker is defined by "body = rocker". Rocker is connected to ground ("inb = \$ground") by a pin joint. This is followed by mass and inertia properties of rocker. The vector from mass center of rocker to the joint is given by "bodytojoint = 0 0 -1". Vector from the origin of ground coordinate system, to rocker pin joint is given by "inbtojoint = 0 1.5 0". The direction of axis of the pin joint is given by "pin = 1 0 0".

The connecting rod ("body = connect") is connected to rocker ("inb = rocker") by a pin joint. Since rocker has already been described above, it can be used as an inboard body. The description of mass and inertia properties of the connecting rod are self explanatory. The statements "bodytojoint = 0 1.5 0", "inbtojoint = 0 0 1", "pin = 1 0 0" are also self explanatory.

Description of the loop joint is given next. Loop joint is between connecting rod and crank. The loop joint may be placed between rocker and connecting rod, depending on user's convenience. For this example description of loop joint begins by the statement "body = connect". For loop joint connecting rod is connected to crank ("inb = crank"). Since description of connecting rod has already been done there is no need of specifying the mass and inertia properties of the body again. The vector from center of mass of connecting rod to the joint is given by "bodytojoint = 0 -1.5 0", and the vector from mass center of crank to the

joint is given by "inbtojoint = 0 0 0.5". The direction of the axis of the pin is specified by "pin = 1 0 0". Thus description of loop joint is similar to the description of the rest of the joints except that both the body and inboard body should have been described before, also no mass and inertia properties are described in it.

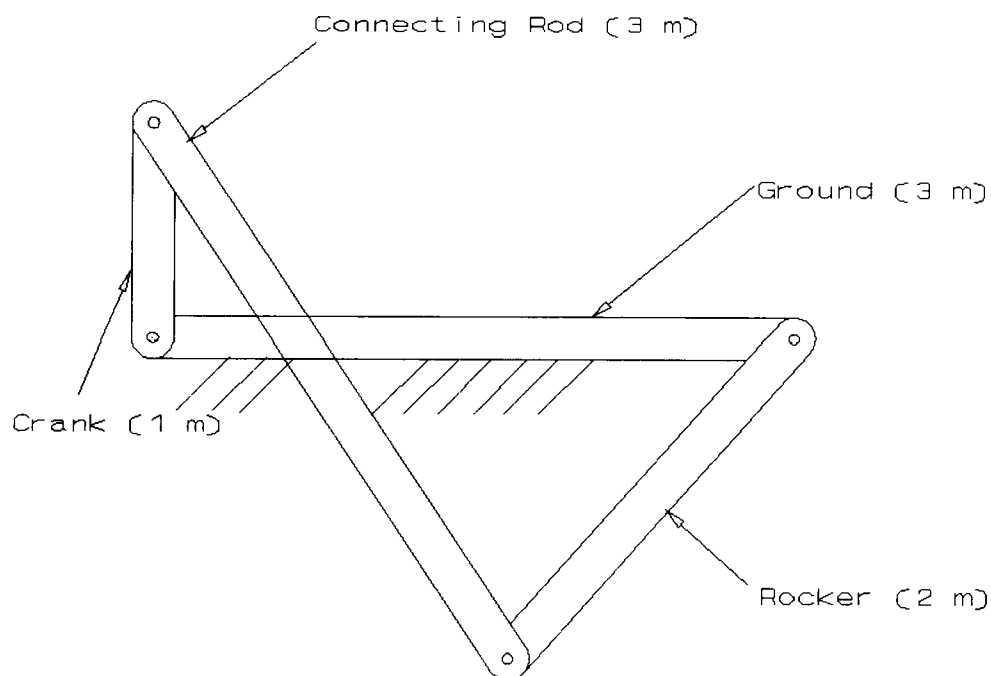


Figure 3.5 : Four-Bar mechanism assembled in one of its geometric configuration.

The user is ready to run SD/FAST after having written the system description file. As explained above SD/FAST generates three files depending on the name of the input file, which in this case would be fbar_i, fbar_d.c and fbar_s.c. User may also want to generate the library file

(sdlib.c). Since the language option ("language = c") was placed in the preamble, SD/FAST generates the code in C, and not in FORTRAN which is the default option for SD/FAST. The user can now write code for doing the simulation wanted.

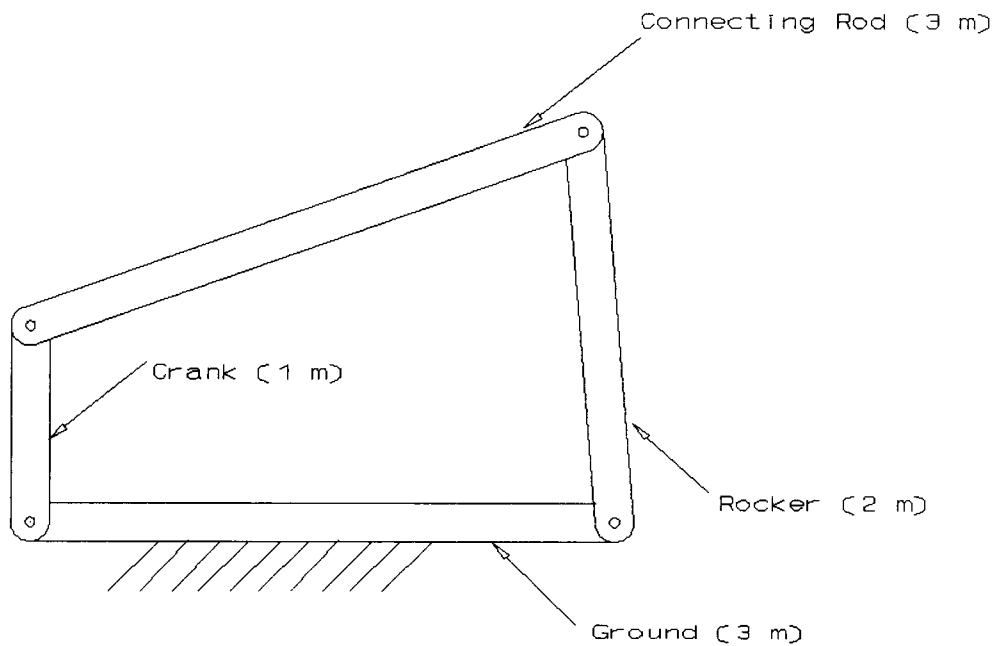


Figure 3.6 : Four-Bar mechanism assembled in its second geometric configuration.

For this example assembly of the mechanism is left for SD/FAST. The system can be assembled by keeping any of the joint angles constant. SD/FAST then assembles the mechanism by varying the other two angles till it finds a solution. As shown in Figure 3.5 and Figure 3.6, SD/FAST may assemble the system in either of two geometric configurations. To overcome this difficulty user can specify initial guesses for the angles and SD/FAST then searches for a solution near these angles.

Figure 3.6 shows the mechanism in which angle of crank with respect to ground is fixed and other two angles, that is, joint angle between connecting rod and rocker and joint angle between rocker and ground are allowed to change. Initial guesses for the angles are given to make the mechanism assemble in the required configuration shown in Figure 3.6.

3.3 Description of Machine

For doing any analysis on walking machine using SD/FAST, the first step is to make system description file for the machine. The kinematic parameters for the machine have been explained in Chapter 2. A-model and D-H kinematic parameters of the walking machine have already been determined. Conversion of A-model kinematic parameters of the machine to the model as accepted by SD/FAST was done. The conversion was done keeping in mind that there might be some changes required later. The preparation of system description file for the machine is explained below and the complete system description file of the machine is attached in Appendix.

As explained in Chapter 2 each leg of the machine consists of three parts, which are coxa, femur and tibia. The joints of body-coxa, coxa-femur and femur-tibia are all revolute and the joint between tibia and ground is a passive ball-and-socket joint. This configuration is similar to a RRR manipulator.

System description file for the machine was initially made with the A-model physical parameters. The location of mass center of the legs was initially done at the middle of the respective leg segment. The inertia matrix was arbitrarily chosen. The mass and inertia properties of the legs were later calculated by making a spreadsheet in Quattro and is attached in the Appendix. The system description file was then modified to account for location of center of mass and inertia matrix of segments of leg. Notice that most of the numbers in the system description file are followed by a "?" mark which means that these numbers can be changed when the simulation program is run.

The set of units used in system description file are consistent with the units used for describing the physical parameters in the A-model. The unit of mass is grams, unit of length is mm and unit of time is seconds for describing the machine in system description file. The unit of angle is chosen by SD/FAST and is radian.

System description file of the machine begins with preamble, which contains information about gravity and programming language. The gravity vector is given by "gravity = 0 0 -9800", which means that the gravity is acting along the negative z_0 . The units of gravity used are g.mm/s².

3.3.1 Description of Body of Machine

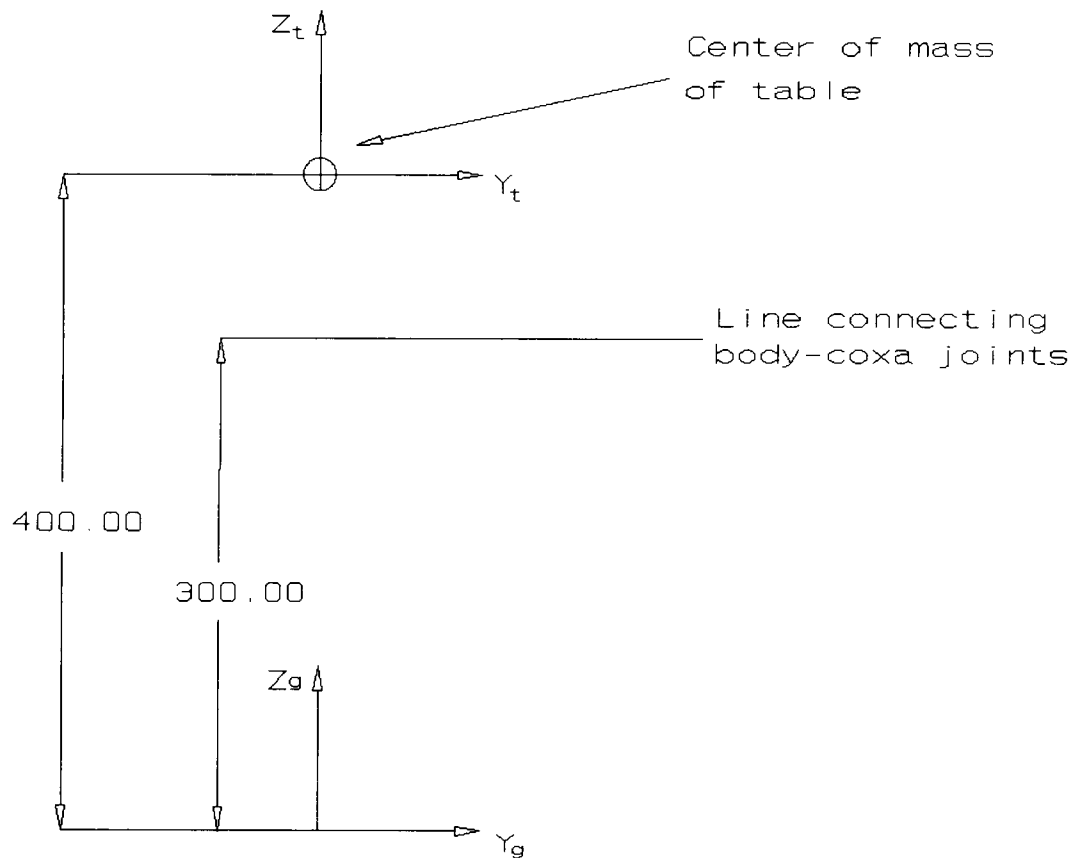


Figure 3.7 : Right view of table, at mass center height of table at 400 mm.

The description of machine begins by describing main body, which is named as table ("body = table"). Figure 3.7 shows the right view of the table with yz_g as the ground coordinate system and yz_t as the table coordinate system. For the system description file the center of mass of table is at a height of 400 mm above ground (xy_g plane) and is placed exactly above the origin of ground coordinate system. Table is connected to ground ("inb = \$ground") by a six degree of freedom joint ("joint =

sixdof") located at center of mass of the table. The mass of machine was measured after manufacturing the basic structure.

Inertia matrix of the machine was calculated by using the physical dimension of the structure and the calculations are shown in Appendix. Vector from center of mass of table to the joint is given by "bodytojoint = 0 0 0". The vector from origin of ground coordinate system to the joint is given by "inbtojoint = 0 0 400". The three pin vectors define the sliding axes fixed on the inboard coordinate frame, that is, ground. The phrase "prescribed = 0? 0? 0? 0? 0? 0?" means that at present the prescribed motion of the joint is off, but later it can be turned on by the user in the code. The motion of the joint axes can be prescribed, usually as a function of time and in general as a function of time and some system state, such as acceleration, velocity and position. Routines are available for providing desired acceleration, velocity and position, with SD/FAST, for prescribed axes. If the "=" appears after prescribed, the number of "1" or "0" following it should be equal to the number of degrees of freedom provided by the joint being defined.

3.3.2 Description of Right Front Leg

Figure 3.8 shows the right front leg as described by A-model. As explained earlier A-Model uses two coordinate systems for each segment,

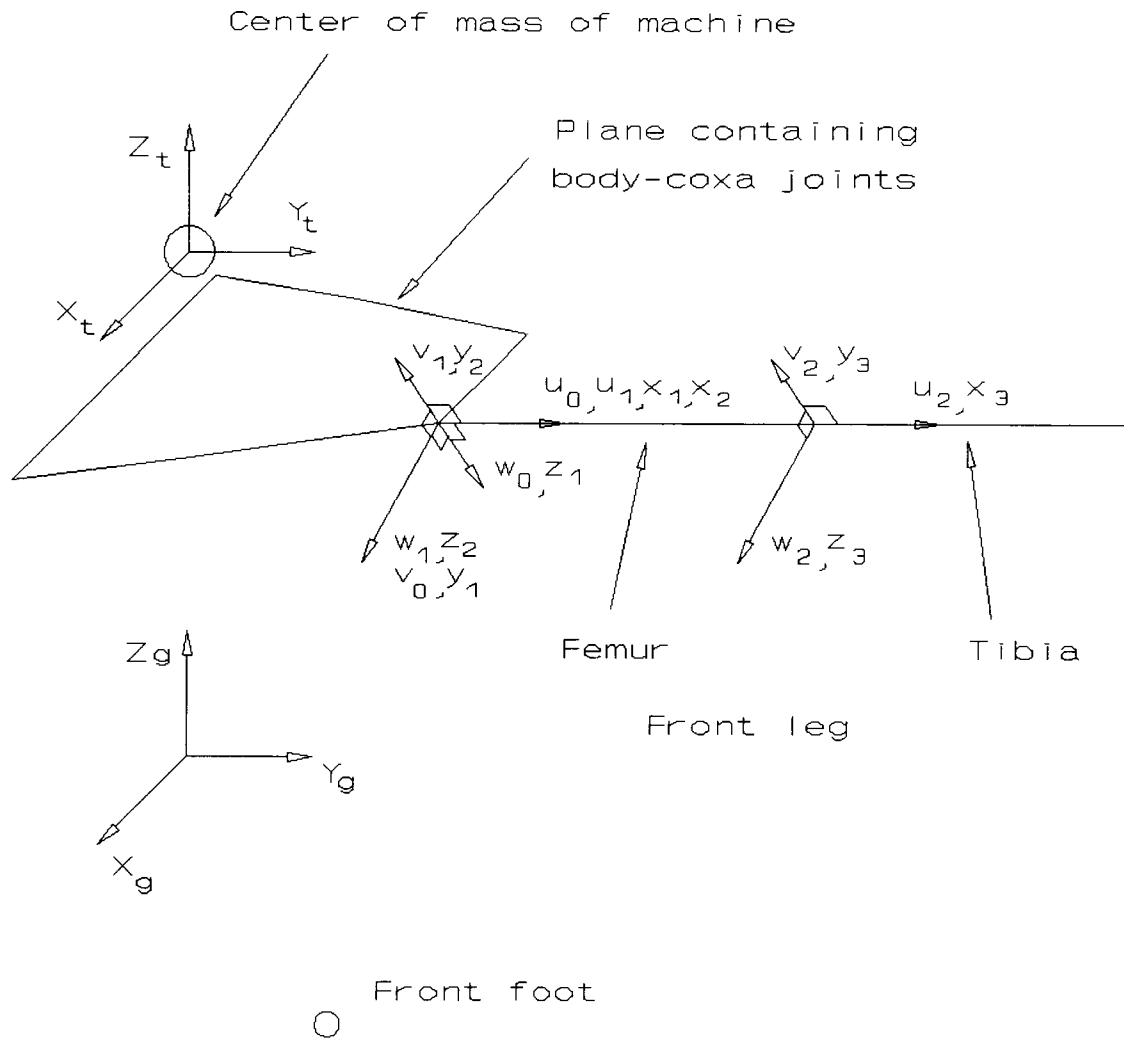


Figure 3.8 : Right Front leg of machine as described by A-Model.

which are xyz_i and uvw_i . It was also explained earlier that for a revolute-revolute pair of joints the uvw_{i-1} coordinate frame coincide with xyz_i coordinate frame. In Figure 3.8 xyz_g represents the ground coordinate system; xyz_t and uvw_0 are the coordinate systems for table; xyz_1 and uvw_1 are the coordinate systems for coxa; xyz_2 and uvw_2 are the coordinate systems for femur and xyz_3 is the coordinate systems for tibia.

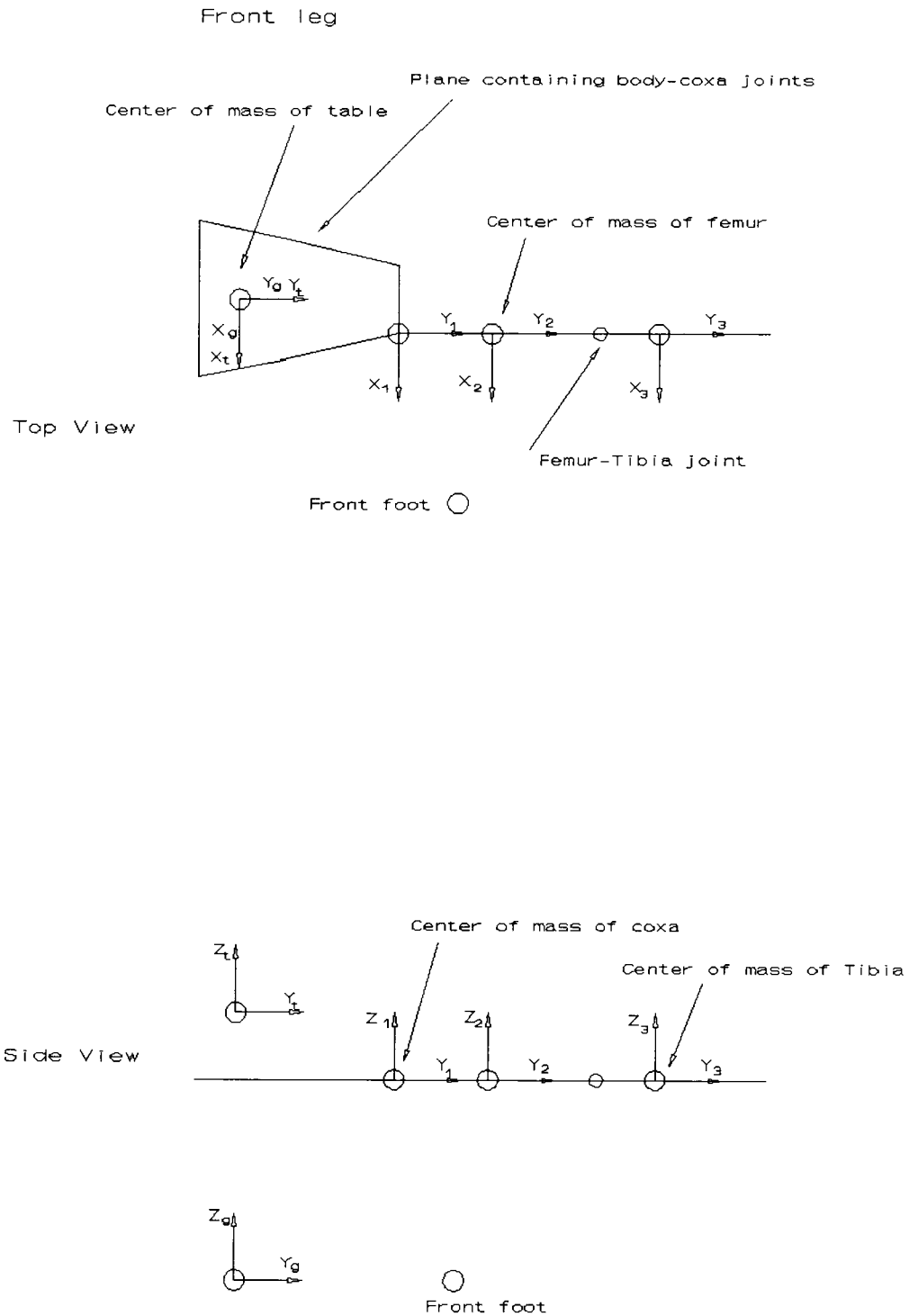


Figure 3.9 : Top and Side view of right front leg of machine, as described for developing system description file for SD/FAST.

The frames uvw_0 and xyz_1 coincide; frames uvw_1 and xyz_2 coincide and frames uvw_2 and xyz_3 coincide. In Figure 3.8 z_1 , z_2 and z_3 gives the axis of rotation of body-coxa, coxa-femur and femur-tibia joints respectively, which are same as w_0 , w_1 and w_2 axis.

Figure 3.9 shows the same leg made for writing the system description file for SD/FAST. In Figure 3.9 xyz_t is the table coordinate frame; xyz_g is the ground coordinate frame; xyz_1 is the coordinate system for coxa and is located at the body-coxa joint as the coxa length is zero; xyz_2 is the coordinate system for femur and is located at the center of mass of femur; xyz_3 is the coordinate system for tibia and is located at the mass center of tibia. The axes of the joints are same as defined by A-Model. The figure shows the table at a height of 400 mm above ground.

The description of right front leg starts with the description of the coxa. The Right FRont COXA ("body = rfrcoxa") is connected to the table ("inb = table") of the machine by a revolute or pin joint ("joint = pin"). The mass and inertia properties of coxa is taken to be equal to the mass and inertia properties of the pulley. Since the coxa is taken to be zero length the mass center of coxa is at the joint ("bodytojoint = 0 0 0"). The vector from center of mass of table to the coxa is given by "inbtojoint = 50 235 -100", as the coxa are placed 100 mm below the center of mass of table as shown in Figure 3.9. The "?" marks after the numerical values in the phrase

"inbtojoint =" of the system description file of machine, will help to change the location of the body-coxa joints in the code written later. The axis of the revolute joint is the same as that of the A-model ("pin = -0.5 0 -0.866"). The phrase "prescribed = 0?" means that at present the prescribed motion of the joint is off, but later it can be turned on by the user in the code.

The Right FRont FEMUR ("body = rirrfemure") is attached to the Right FRont COXA ("inb = rirfcoxa") by a revolute joint ("joint = pin"). The mass and the inertia properties were calculated taking into account the physical properties of the leg, and the calculations are attached in the Appendix. The location of coordinate system for the femur was done at the mass center located at the point where the inertias along the axes were found to be minimum. The vector from the mass center of femur to the joint is given by "bodytojoint = 0 -139 0", and the vector from the mass center of coxa to the joint is given by "inbtojoint = 0 0 0", as the center of mass of coxa is located at the joint. The axes of the pin is defined by "pin = 0.866 0 -0.5", and the prescribed motion is left off "prescribed = 0?" to be used later in the code written.

The body of the machine is connected to the femur by two joints which are perpendicular to each other, and are equivalent to a Hooke joint. In the system description file the connection is not modelled as a Hooke joint

because the length of the coxa could be changed without making appreciable changes to the file. Also breaking the whole body of the machine in more parts makes it easier to describe.

The Right FRont TIBIA ("body = rifrtibia") is attached to ("inb = rifrfemure") by a pin joint ("joint = pin"). The mass and inertia properties are taken from the calculations done in the spreadsheet, attached in Appendix. Vector from the mass center of tibia to the joint is given by "bodytojoint = 0 -87 0", and the vector from the center of mass of the femur to the joint is given by "inbtojoint = 0 161 0". The direction of the axis of the joint is calculated from the parameters given in the A-model, and is given by "pin = 0.866 0 -0.5". The prescribed motion is turned off and will be used later in code developed.

The loop joint is between the tibia and ground. The Right FRont TIBIA ("body = rifrtibia") is connected to ground ("inb = \$ground") by a ball-and-socket joint ("joint = ball"). The vector from the center of mass of the tibia to the joint is given by the "bodytojoint = 0 163 0", and the location of the foot of the right front leg (ball-and-socket joint) in the ground coordinate system is given by "inbtojoint = 300 325 0". For SD/FAST the motion of ball joint is always modeled as a relationship between local coordinate frames of the two connected bodies. Ball joints are represented by quaternions (Euler parameters), which express the relative orientation of the

outboard body's local frame with respect to the inboard body's local frame. The direction of the axes of the ball-and-socket joint is not required by SD/FAST. If the user wants to give three axes of the ball-and-socket joints, the user should use gimbal joint. The prescribed motion for the three degrees of freedom of the joint are left off, which may be used later.

3.3.3 Description of Right Middle and Right Hind Leg

The description of right middle and right hind legs are very similar to the description of right front leg. For describing the right middle leg, the leg was oriented at an angle of thirty degrees from the yz_g plane. Figure 3.10 shows the right middle leg of the machine, along with the segments of the legs, as drawn for system description file. For the figure the coordinate systems are numbered the same as was done for the front leg in Figure 3.9. The other main difference of the right middle leg from the right front leg is the length of the femur and tibia. Coxa length is zero for the middle leg, but the lengths of femur and tibia are larger than that of the front leg. The mass and inertia properties for the segments are also greater and the calculations of the inertia matrix is shown in Appendix. The difference is also in the location of the body-coxa joint and the location of the tibia-ground joint or foot position.

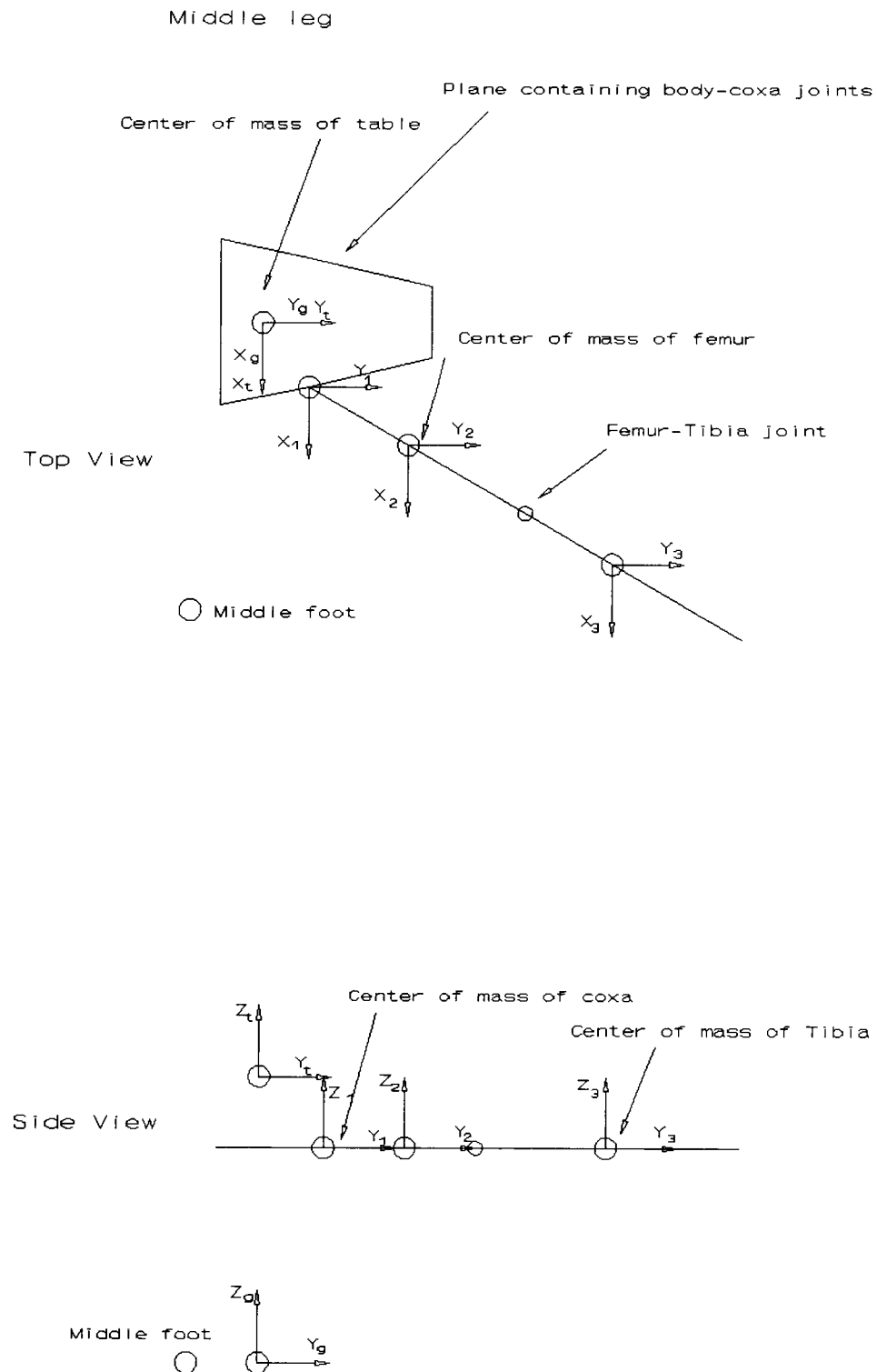


Figure 3.10 : Top and Side view of right middle leg of machine, as described for developing system description file for SD/FAST.

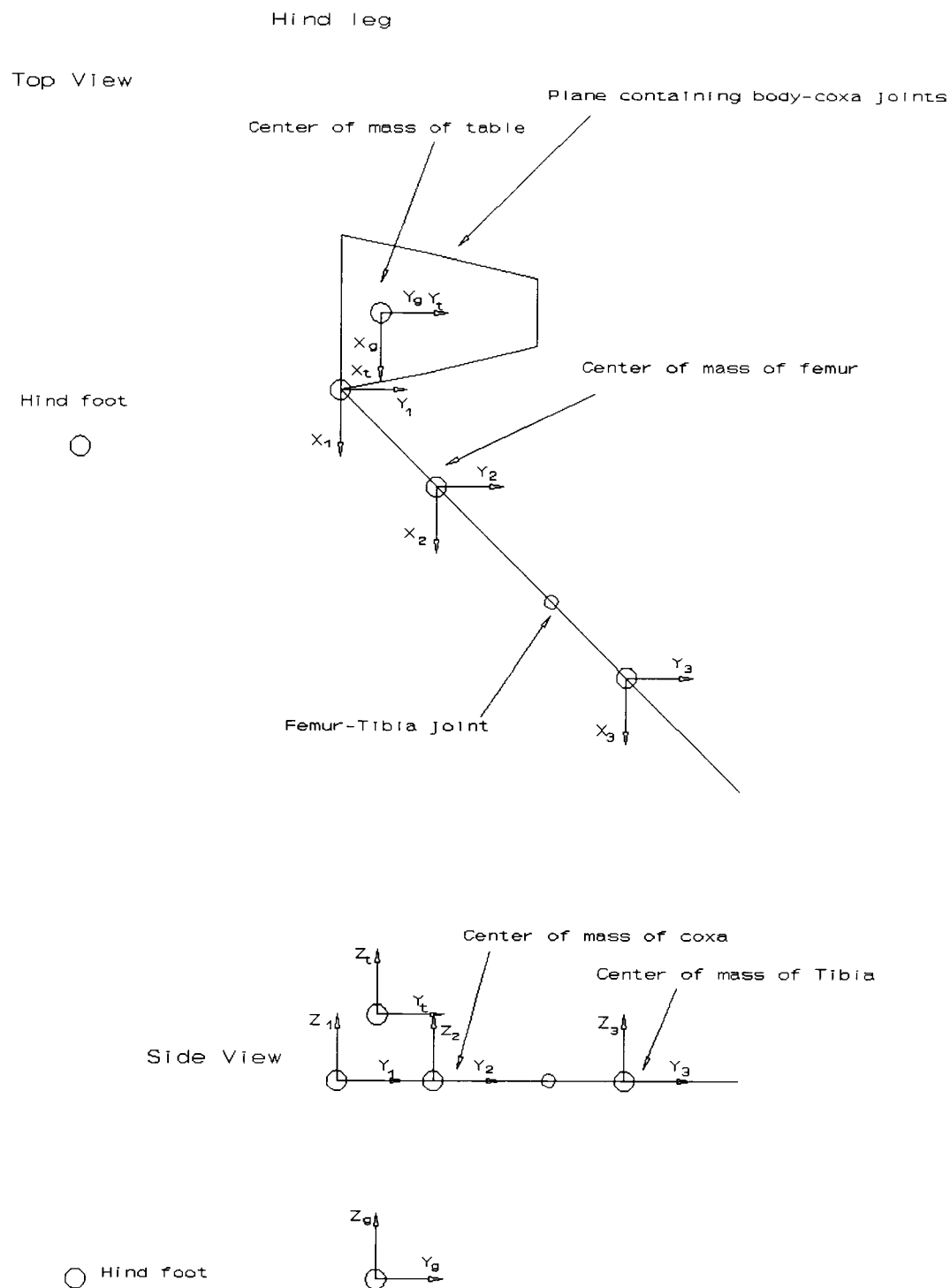


Figure 3.11 : Top and Side view of right hind leg of machine, as described for developing system description file for SD/FAST.

The right hind leg is oriented at an angle of forty-five degrees from the yz_g plane. As compared to the right front leg the difference is in the length and mass of the segments of legs, the position of the body-coxa joints and tibia-ground joint. Figure 3.11 shows the right hind leg of the machine as drawn for system description file using A-Model parameters. In the figure coordinate systems are numbered the same way as for Figure 3.9.

3.3.4 Description of Left Legs

The description of the left legs is similar to the right legs. For describing the left legs use of symmetry is made. Since the whole body is symmetrical about the yz_g plane of the machine coordinate system, passing through the center of mass of the body, the description of the left legs is very simple, once the description of the right legs have been done. The only changes made in the description of the left legs is that of changing the sign of all the X coordinates of vectors. For example :

For describing right front coxa the "inbtojoint = 50 235 -100" was used, but for describing the left front coxa "inbtojoint = -50 235 -100" is used. For describing the direction of the revolute joint between right front femur and right front tibia use of "pin = 0.866 0 -0.5" was done, but for describing the direction of the same joint for the left legs use of "pin = -0.866 0 -0.5" is done.

The mass and the inertia properties of the legs is not changed. Also length of segments of the left legs is the same as those of the right legs. The position of foot, that is, the ball and socket joint between tibia and ground is also the same except the change in the sign of X coordinate in "inbtojoint = " phrase.

After having developed system description file for the machine, next step is to generate the code by using SD/FAST. Since the system description file is long SD/FAST generates nine files, which are mac_i, mac_s.c, mac_d.c, mac_d00.c, mac_d01.c, mac_d02.c, mac_d03.c, mac_d04.c and mac_d05.c. For doing any kind of simulation, along with above files sdlib.c and user written code is needed to create an executable file.

4. Body Workspace

4.1 Kinematic Workspace

Kinematic Workspace is defined as volume in space where center of mass of machine can be placed so that all three joint angles of all six legs are within their specified ranges. Kinematic workspace of the machine is important to study as it gives an idea of the positions where the machine can reach without crossing any of the maximum and minimum values of the joint angles. The maximum and minimum values of the joint angles are to a certain extent also dependent on the physical dimensions of the segments of legs. Figure 4.1 shows the machine in assembled position with mass center height of machine at 400 mm, for a set of foot positions.

Let Θ_1 represent the joint angle between body and coxa, Θ_2 the joint angle between coxa and femur and Θ_3 the joint angle between femur and tibia, then a point is said to be within kinematic workspace only when joint angles of all the six legs satisfy equation 4.1, 4.2 and 4.3.

$$(\Theta_1)_{\min} \leq \Theta_1 \leq (\Theta_1)_{\max} \quad \text{..(4.1)}$$

$$(\Theta_2)_{\min} \leq \Theta_2 \leq (\Theta_2)_{\max} \quad \text{..(4.2)}$$

$$(\Theta_3)_{\min} \leq \Theta_3 \leq (\Theta_3)_{\max} \quad \text{..(4.3)}$$

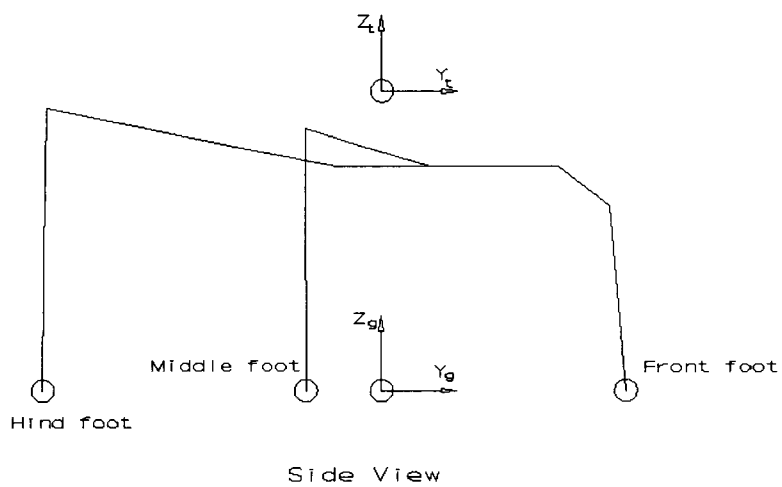
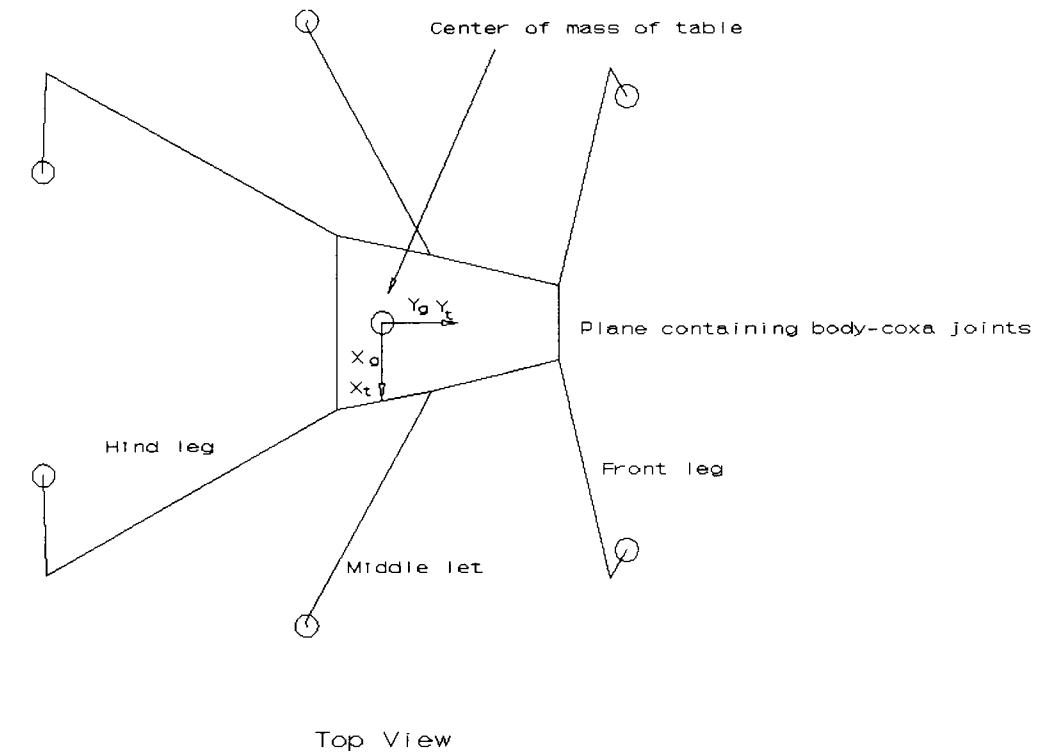
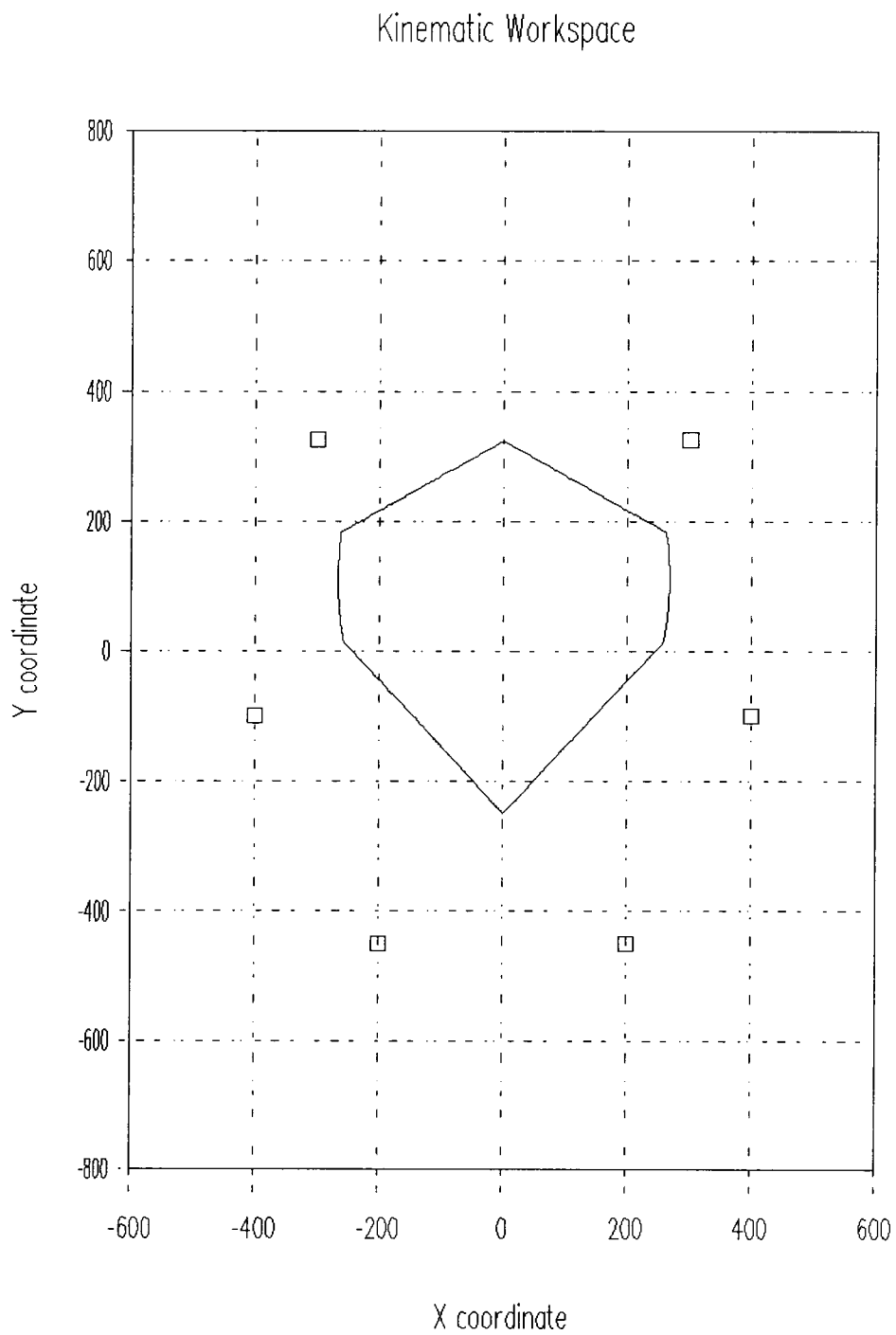


Figure 4.1 : Top and Side view of machine in assembled position, as done by SD/FAST.

The maximum and minimum values of joints angles, that is $(\Theta_i)_{\min}$ and $(\Theta_i)_{\max}$, of right legs of the machine are specified in Table 2.2. As shown in Table 2.1 these values of joint angles for beetle are similar to those of machine. For fixed foot positions, kinematic workspace of machine can be found by mapping the mass center of the machine on plane parallel to xy_g plane, that is, height of center of mass of machine. For finding kinematic workspace code was written in C and linked with the code generated by SD/FAST. Since SD/FAST has capabilities of assembling the system, the assembly of the machine was left to SD/FAST. Initial guesses for the joint angles were passed for assembly, to make sure that the system is assembled in the required configuration. After the assembly has been performed by SD/FAST check on joint angles were made to satisfy equation 4.1, 4.2 and 4.3 for each leg to determine whether a point is within kinematic workspace or not. This procedure was followed for each possible position.

Kinematic workspace of machine depends on foot positions, pitch of body (rotation about X axis of body coordinate system), roll, yaw, height of center of mass of body and several other parameters. In the study the variation of kinematic workspace of machine is investigated with variation of pitch, foot positions and body height. The study of kinematic workspace of machine is not done with variation of roll and yaw, since roll and yaw make the machine asymmetric about the yz_g plane. At this moment only



Pitch = -10, Height = 300, P = 10

Figure 4.2 : Kinematic workspace of machine, at pitch of -10 degrees, foot position 10 and height of center of mass of table at 300 mm.

symmetrical disposition of machine are considered, since the first main objective of the project is to make the machine stand and move it in a straight line parallel to y_g axis.

Figure 4.2 shows kinematic workspace of machine for a particular set of parameters. The square boxes represent one set of foot positions, and the contour is the boundary of the kinematic workspace of machine when the height of center of mass of machine is 300 mm above ground and pitch of body is -10 degrees. For this set of parameters the kinematic workspace of the machine was found by doing grid search with grid size of 4 mm in both x_g and y_g direction, that is, the movement of the machine was started at -400 mm along y_g axis in ground coordinate system and moved along positive y_g axis. As soon a solution is found on y_g axis, the body is moved along positive x_g axis to see if a solution exist to satisfy equation 4.1, 4.2 and 4.3. If a solution exists the body is moved further along x_g axis, till the end of kinematic workspace is reached at a particular coordinate in y_g . After reaching the limit along x_g axis, the center of mass of machine is moved along y_g axis, and the above steps are repeated till solution along y_g axis no longer exists. Since the body and foot positions are symmetrical about the yz_g plane the search is not performed along negative x_g axis, since the workspace will also be symmetrical.

4.2 Force Workspace

As discussed earlier and also in system description file of machine, the joint between tibia and ground is a passive ball-and-socket joint. This joint simulates a point contact between ground and tibia. As a result there cannot be a tensile force between ground and any of the legs, that is, the force that the leg exerts on ground cannot be upwards. If the forces exerted by the legs on the ground is to be upwards, the joint between tibia and ground should be fixed or bolted to the ground, which would mean that the machine cannot move. Thus the forces exerted by the legs on the ground should be downwards, or the forces exerted by the legs on the body should be upwards, that is the forces exerted by the legs on the body should be such that they counter balance the body weight.

If F_1 , F_2 and F_3 be the forces exerted on body by right front, right middle and right hind legs respectively and F_4 , F_5 and F_6 be the forces exerted on body by left front, left middle and left hind legs respectively then a position of center of mass of machine is said to be in force workspace only when equation 4.4, 4.5, 4.6, 4.7, 4.8 and 4.9 are satisfied.

$$F_1 \geq 0.0 \quad \text{..(4.4)}$$

$$F_2 \geq 0.0 \quad \text{..(4.5)}$$

$$F_3 \geq 0.0 \quad \text{..(4.6)}$$

$$F_4 \geq 0.0 \quad \text{..(4.7)}$$

$$F_5 \geq 0.0 \quad \text{..(4.8)}$$

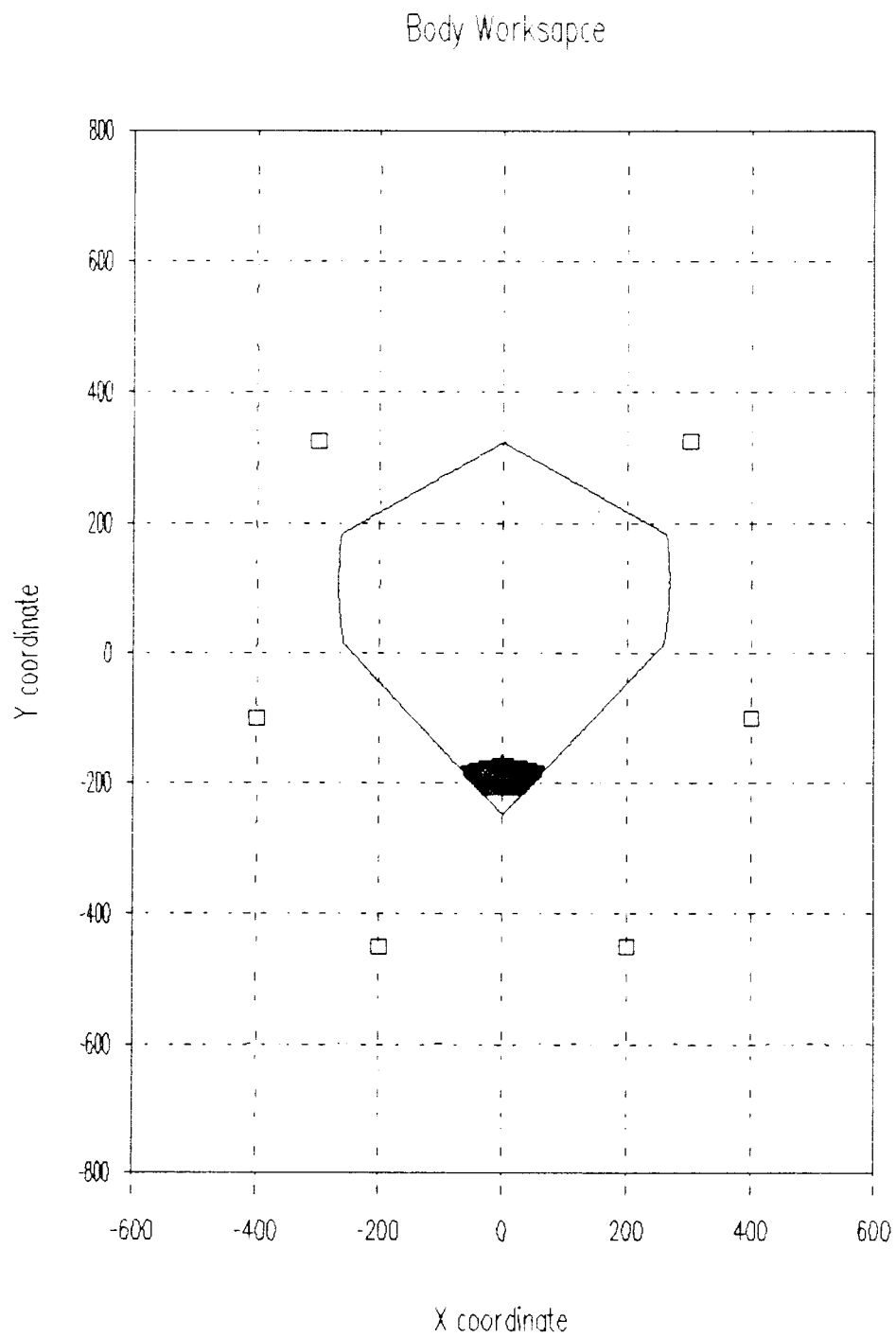
$$F_6 \geq 0.0 \quad \text{..(4.9)}$$

The forces in all the legs should be such that they counter balance the body weight. Center of mass of machine can be mapped on a plane parallel to xy_g plane, that is height of center of mass of machine, and forces in the legs can be calculated to find force workspace. For calculating the forces that the legs exert on body of machine, prescribed motion for femur-tibia joints was turned on, and the acceleration and velocity at the joints was set to zero, thus making the whole body static. This gives the forces at the body coxa joints. If there is no torque applied at the body-coxa and coxa-femur joints, and torques are applied at femur-tibia joints only, then the system is equivalent to a wrench support model (Fichter et al 1988). As each leg is equivalent to a wrench support model, the force exerted by the legs will be in the direction from foot to body-coxa joint. These forces calculated by SD/FAST are in the coxa coordinate frame and are transformed to ground coordinate frame using a subroutine generated by SD/FAST. SD/FAST also calculates the torques that need to be applied at the femur-tibia joints to make the body stand.

4.3 Body Workspace

Body workspace is defined as intersection of kinematic workspace and force workspace. A location in space is said to be within body workspace when it satisfies two criteria, which are, firstly all the joint angles should be within their maximum and minimum limits (satisfying equation 4.1, 4.2 and 4.3) and secondly forces in all the legs are compressive (satisfying equation 4.4 through 4.9). Thus body workspace is the volume in space, where the center of mass of machine can be placed such that equation 4.1 through 4.9 are satisfied. Body workspace is thus the location in space where the machine can practically stand by applying torques at femur-tibia joints only, and the joint angles are within their ranges. The mass center of machine can be mapped on the plane parallel to xy_g plane to find the force workspace within kinematic workspace, and thus find body workspace. The procedure followed for finding body workspace is same as the procedure followed for determining kinematic workspace.

Body workspace of machine depends on height, roll, pitch, yaw, foot positions and other parameters. In this study the variation of body workspace of machine is not considered with variation in roll and yaw due to the reason as explained earlier. Figure 4.3 shows body workspace of machine for a particular set of parameters.



Pitch = -10, Height = 300, P = 10

Figure 4.3 : Body workspace of machine, at pitch of -10 degrees, foot position 10 and height of center of mass of table at 300 mm.

In Figure 4.3 square boxes are the foot positions, the contour is kinematic workspace boundary and the black area within the kinematic workspace is the region where forces in all the legs are compressive. As discussed earlier the force workspace is found within the kinematic workspace, thus directly giving body workspace. Thus black area also represent body workspace. For the figure grid search was made with a size of 4 mm in both X and Y axis. The figure shows body workspace of machine for foot position 10, pitch of -10 degrees and body height of 300 mm.

For studying body workspace three foot positions were selected. These foot positions were numbered in the ongoing research on walking machine at OSU. The data for the foot positions 10, 12, 19 are given in Table 4.1, 4.2 and 4.3 respectively. The main reason for selecting these foot positions was to make the body move forward, that is, starting from foot position 10 if front legs are lifted and placed forward then foot position 19 is reached, if then middle legs are lifted and placed forward then foot position 12 is reached and lifting hind legs and placing forward foot position 10 is reached, thus completing a sequence. It can be seen that all the foot positions chosen are symmetric about the yz_g plane and the machine is also symmetrical about yz_g plane, thus for body workspace it is only necessary to satisfy the equations 4.4, 4.5 and 4.6, as the forces in the left legs will be equal in magnitude but different in the direction of only X component.

Table 4.1 : Position of foot in ground coordinate system for foot position 10

	X Coordinate	Y Coordinate	Z Coordinate
Right Front Foot	300	325	0
Right Middle Foot	400	-100	0
Right Hind Foot	200	-450	0
Left Front Foot	-300	325	0
Left Middle Foot	-400	-100	0
Left Hind Foot	-200	-450	0

Table 4.2 : Position of foot in ground coordinate system for foot position 12

	X Coordinate	Y Coordinate	Z Coordinate
Right Front Foot	300	500	0
Right Middle Foot	400	75	0
Right Hind Foot	200	-625	0
Left Front Foot	-300	500	0
Left Middle Foot	-400	75	0
Left Hind Foot	-200	-625	0

Table 4.3 : Position of foot in ground coordinate system for foot position 19

	X Coordinate	Y Coordinate	Z Coordinate
Right Front Foot	300	500	0
Right Middle Foot	400	-275	0
Right Hind Foot	200	-625	0
Left Front Foot	-300	500	0
Left Middle Foot	-400	-275	0
Left Hind Foot	-200	-625	0

It was found during the study that pitch of the body is an important factor affecting the body workspace. Pitch is defined as the rotation of body about the x_b axis of body coordinate system. Variation of body workspace with pitch of 0, -5, -10, -15 and -20 degrees was studied. In the study only negative pitch of the body is considered since it was found that beetle walk with some negative pitch. Figure 4.4 shows the table at pitch

of 0, -10 and -20 degrees and height of center of mass of body at 400 mm. The other important factor affecting the body workspace is height of mass center of body. The body workspace of machine is studied with heights of 350, 300, 250 and 200 mm above ground.

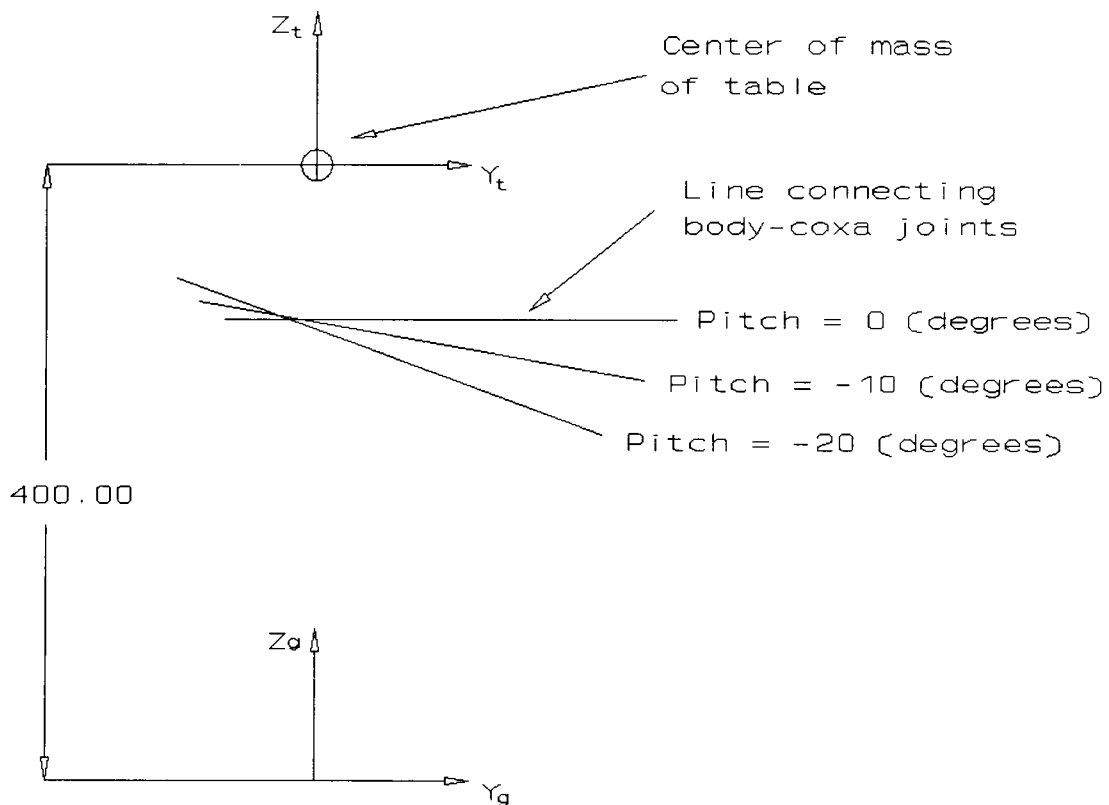


Figure 4.4 : Table with pitch of 0, -10 and -20 degrees.

For studying the variation of body workspace of machine with foot positions, pitch and height of mass center of machine use of procedure explained by Foo et al (unpublished) was made, since the computation performed by the procedure is much faster as compared to computation done by SD/FAST.

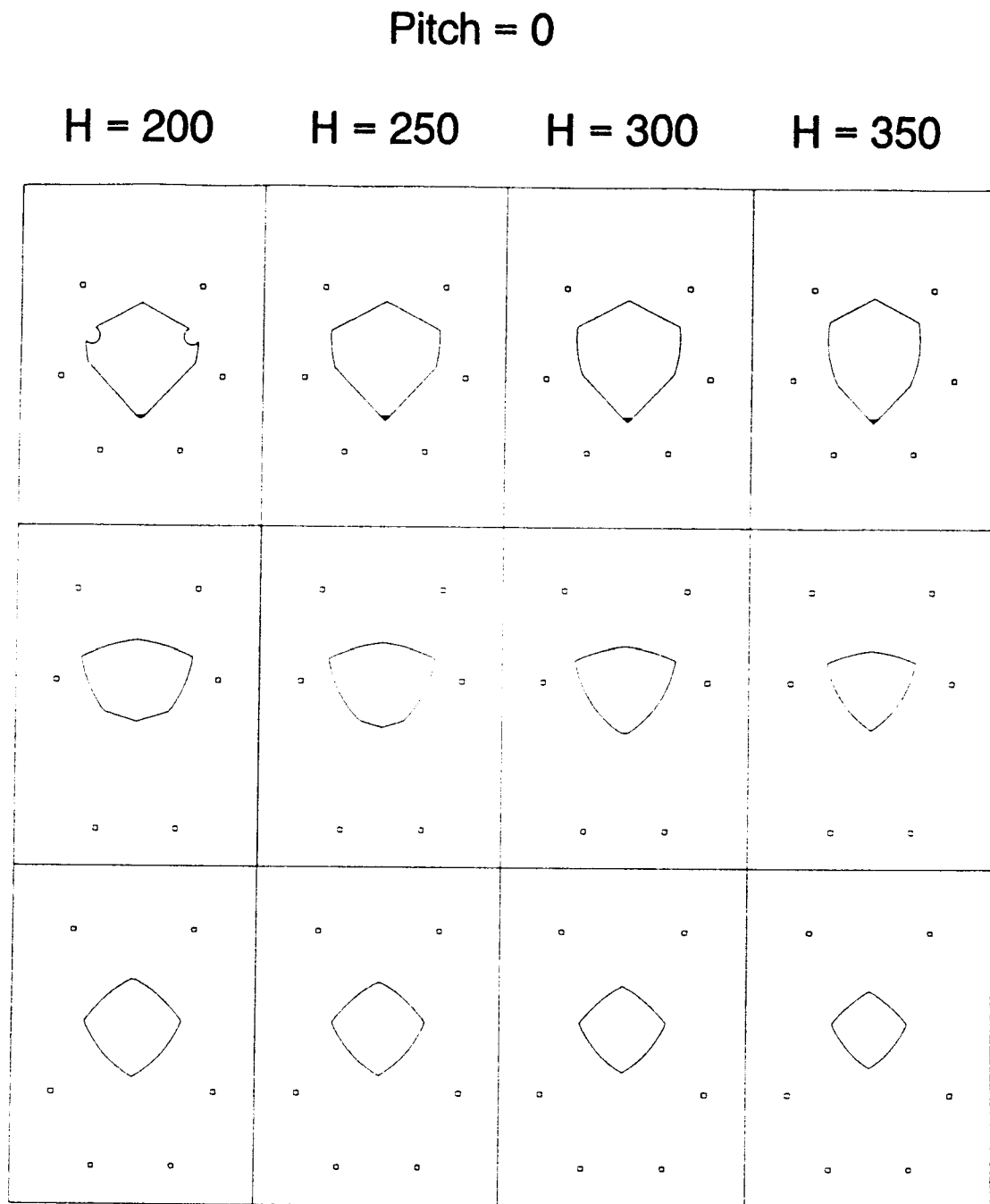


Figure 4.5 : Body workspace of machine at pitch of 0 degrees, with variation in height of mass center of table and foot position.

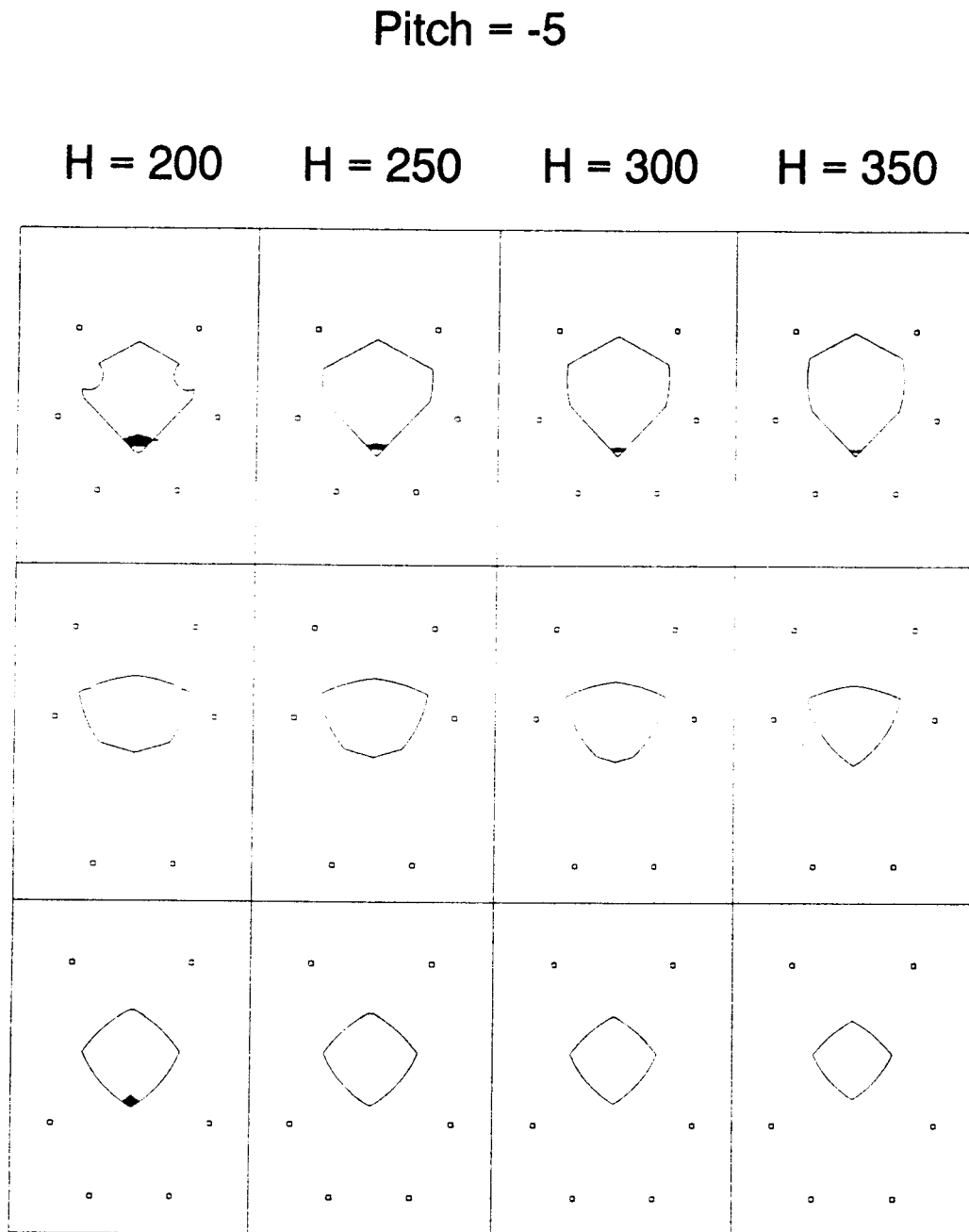


Figure 4.6 : Body workspace of machine at pitch of -5 degrees, with variation in height of mass center of table and foot position.

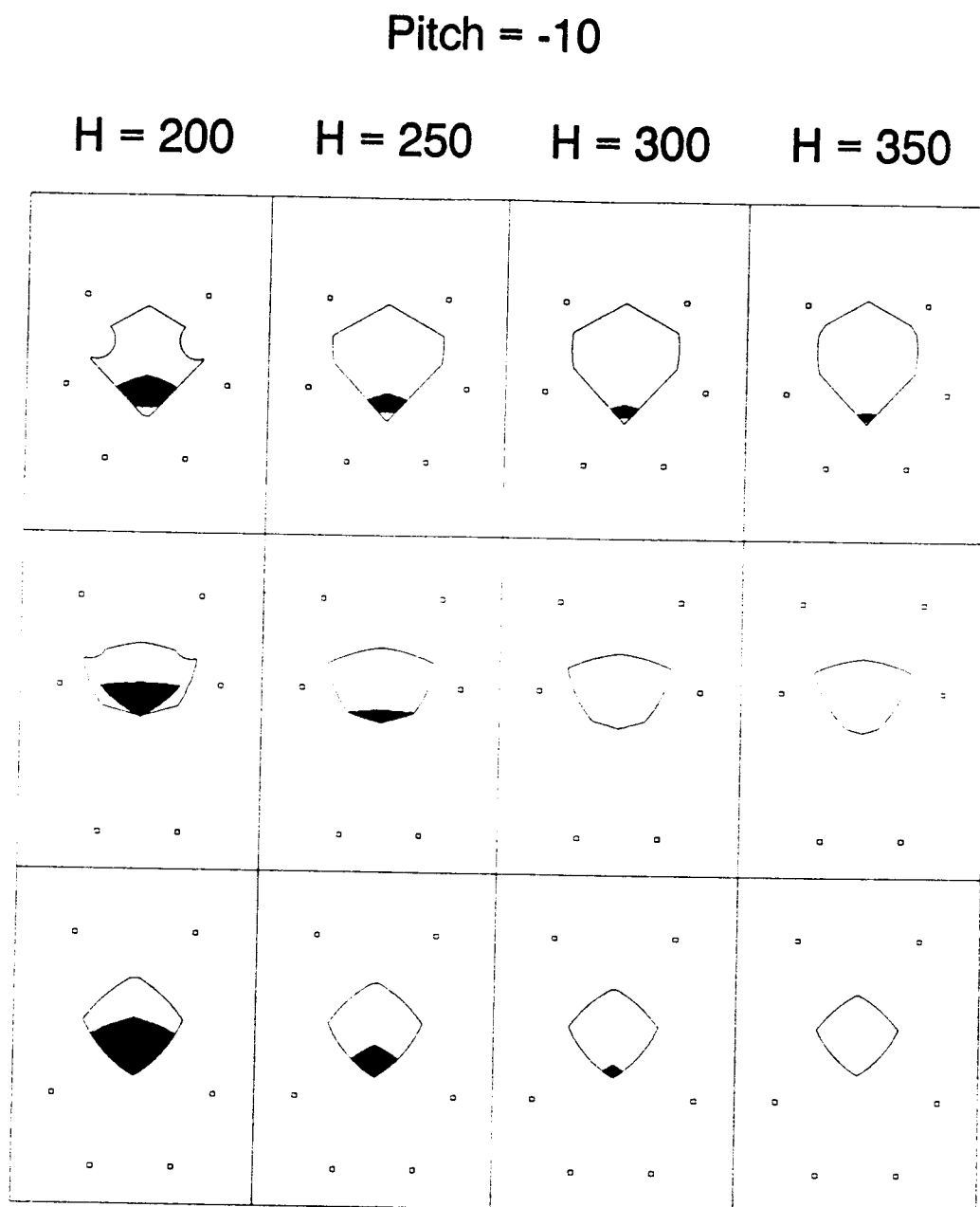


Figure 4.7 : Body workspace of machine at pitch of -10 degrees, with variation in height of mass center of table and foot position.

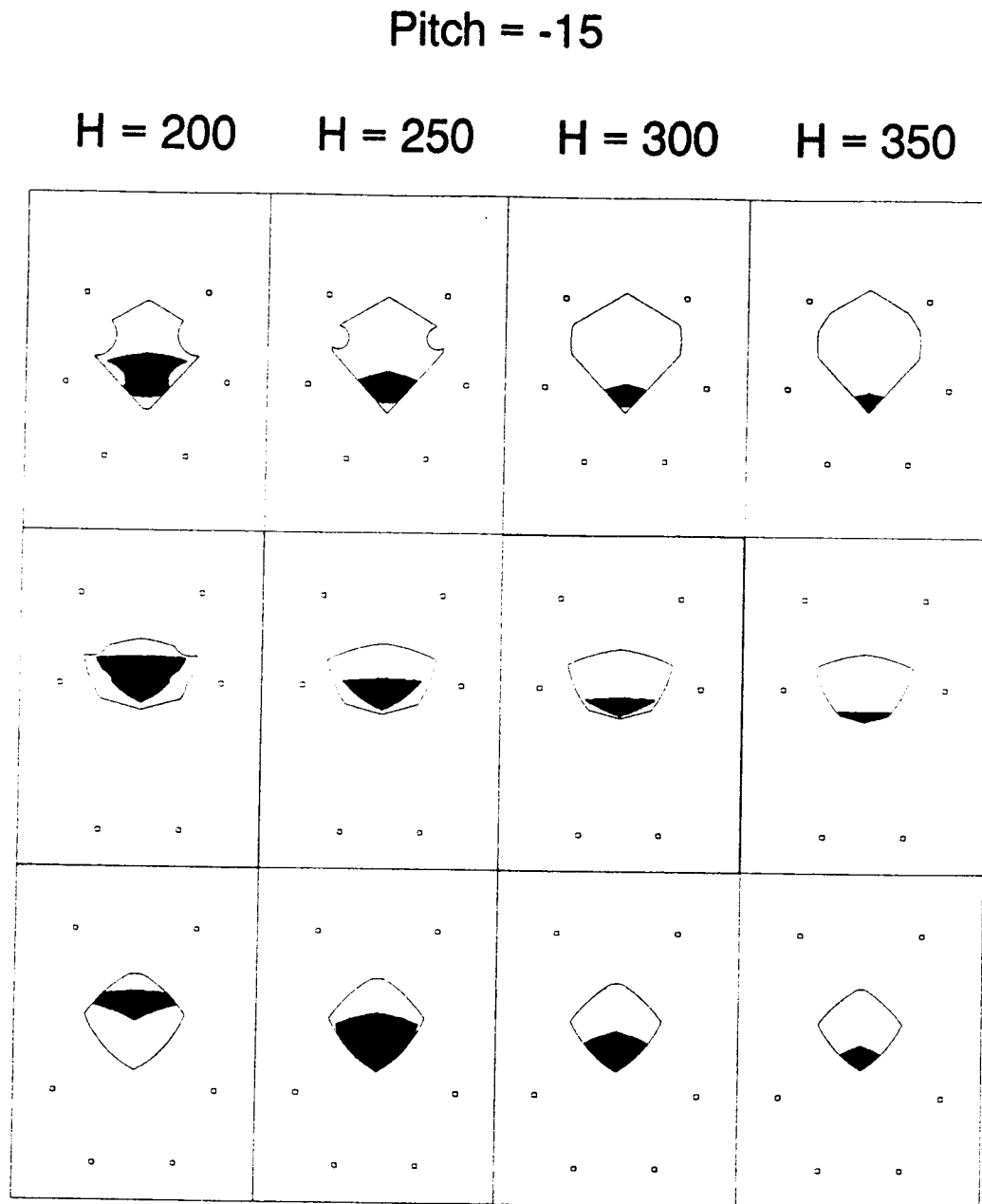


Figure 4.8 : Body workspace of machine at pitch of -15 degrees, with variation in height of mass center of table and foot position.

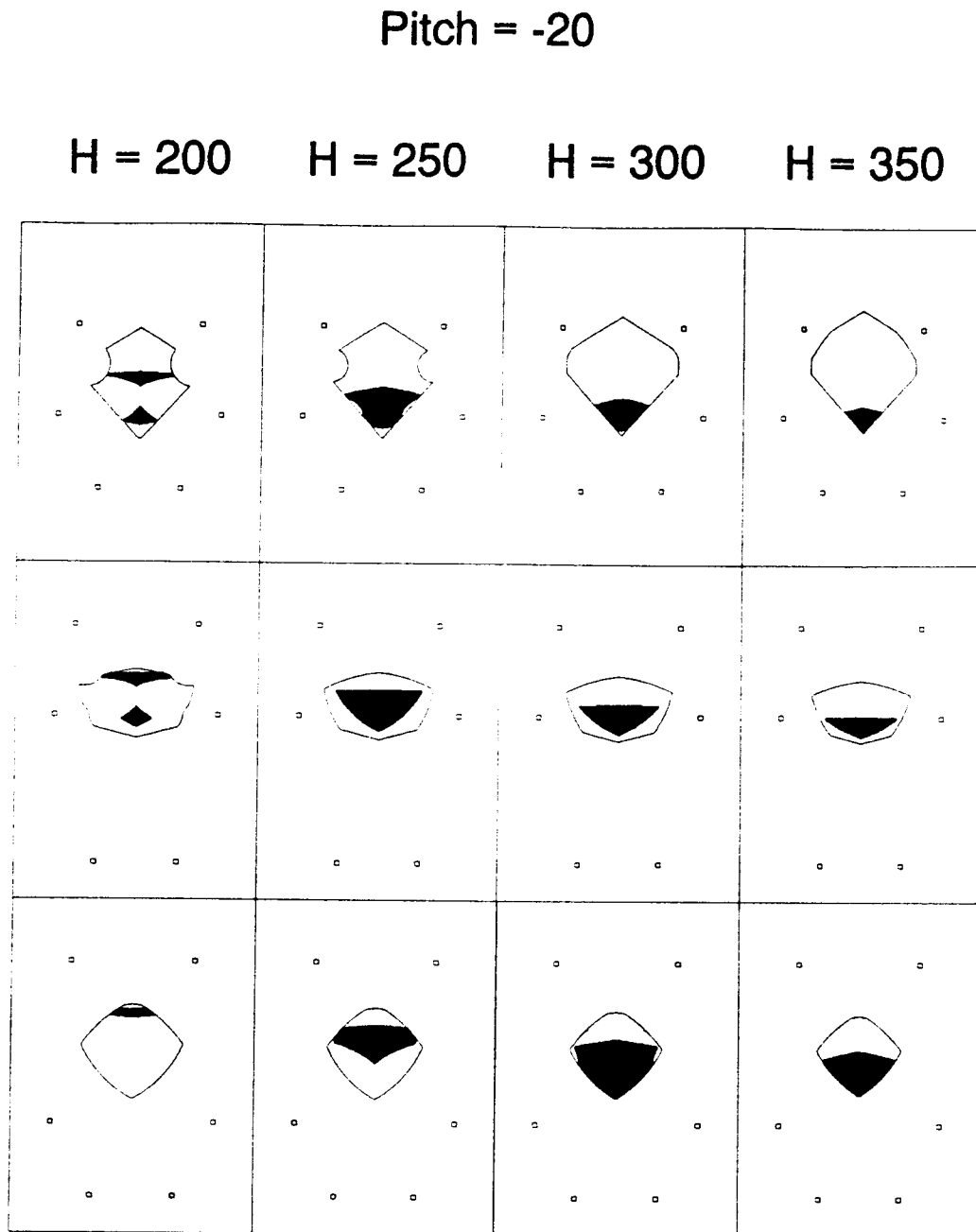


Figure 4.9 : Body workspace of machine at pitch of -20 degrees, with variation in height of mass center of table and foot position.

Figure 4.5, 4.6, 4.7, 4.8 and 4.9 shows the body workspace of the machine with body pitch of 0, -5, -10, -15 and -20 degrees respectively with variation in both body height and foot positions. The figures along the rows are with variation in height of mass center of machine and along the columns are with variation in foot positions. From the figures following conclusions can be drawn :

- (i) The kinematic workspace and force workspace of machine is symmetric about the yz_g plane, or in other words the body workspace of machine is mirror image about yz_g plane. The main reason for the symmetry is because whole body is symmetrical about the yz_g plane.
- (ii) In most of the plots, force workspace is in the lower part of kinematic workspace, unless the force workspace is split.
- (iii) For same height and pitch, foot position 19 has the largest body workspace as compared to body workspace with foot position 10 and 12, except when the workspace is split or the force workspace lies below the kinematic workspace.
- (iv) Increase in magnitude of pitch of machine gives larger body workspace for same foot position and body height.

(v) Increase in height of center of mass of body gives smaller body workspace for same foot positions and pitch of machine.

(vi) Decreasing pitch by 5 degrees and increasing height of body by 50 mm gives similar body workspace, for same foot positions.

From the above conclusion it was seen that there is increase in body workspace of machine with increase in magnitude of pitch and decrease in height of mass center of machine. Since relatively large body workspace of machine, at any foot position, occurs at larger pitch and smaller height, which makes the front body coxa joints very near to ground, which is undesirable.

Increase in body workspace of the machine with increase in magnitude of pitch may be due to decrease in height of body coxa joints of the front legs, or due to increase in the height of body coxa joints of the hind legs, or both. From observation of darkling beetles it appears that front body-coxa joints are lower than middle and hind body-coxa joints. Thus the height of body-coxa joints of front legs was reduced by 50 mm relative to table coordinate system and study of body workspace was done with variation in pitch, height of mass center of machine and foot position.

Figure 4.10 shows the table with pitch of 0, -10 and -20 degrees, and body height of 400 mm and with front body coxa joints down by 50 mm. Figure 4.11, 4.12, 4.13, 4.14, and 4.15 shows the body workspace of the machine with body pitch of 0, -5, -10, -15 and -20 degrees respectively. The figures along the rows are with variation in height of mass center of machine and along the columns are with variation in foot positions.

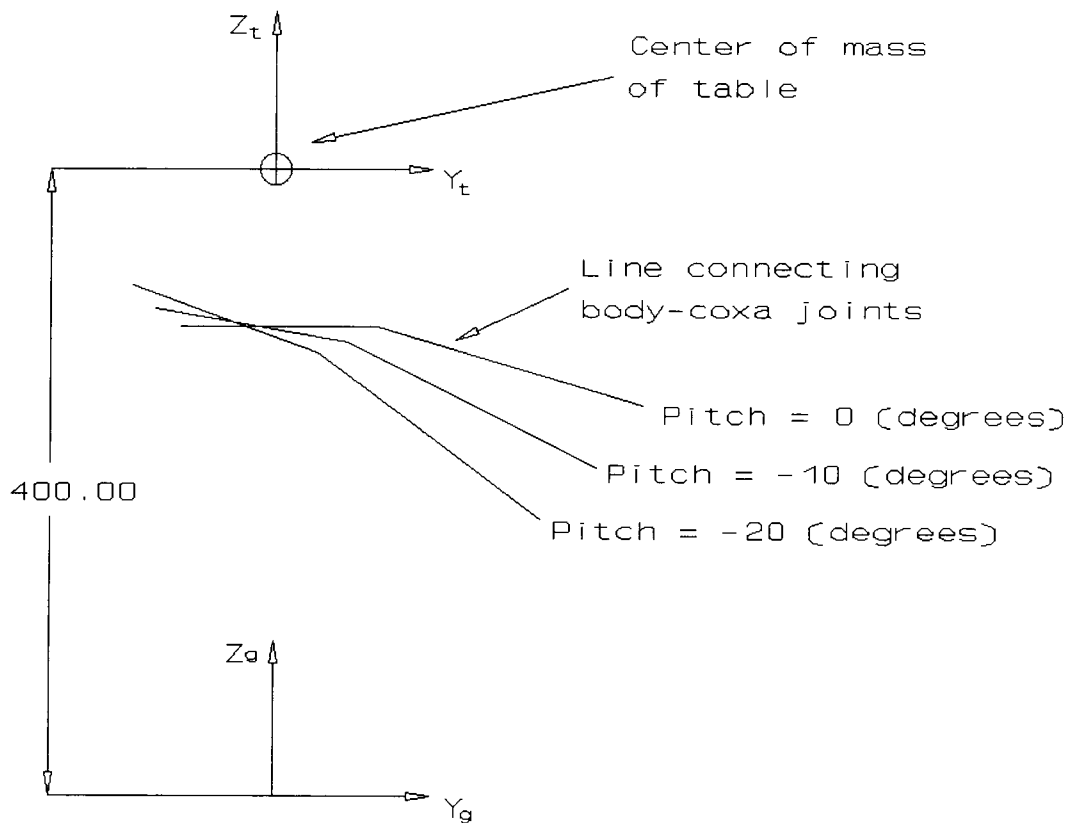


Figure 4.10 : Table with pitch of 0, -10 and -20 degrees, and front body coxa joints below by 50 mm.

Pitch = 0a

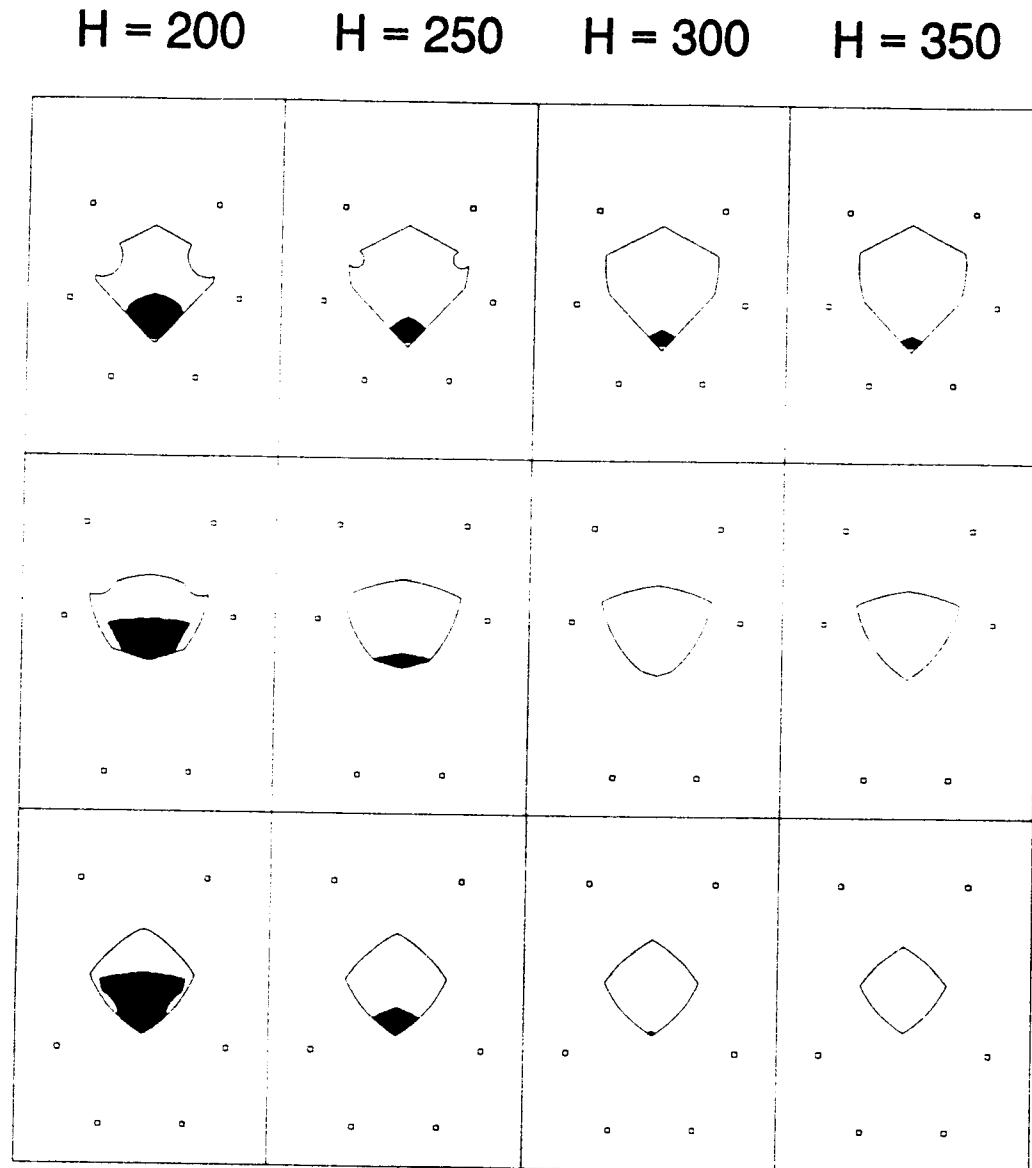


Figure 4.11 : Body workspace of machine at pitch of 0 degrees, with variation in height of mass center of table and foot position, and front body coxa joints below by 50 mm.

Pitch = -5a

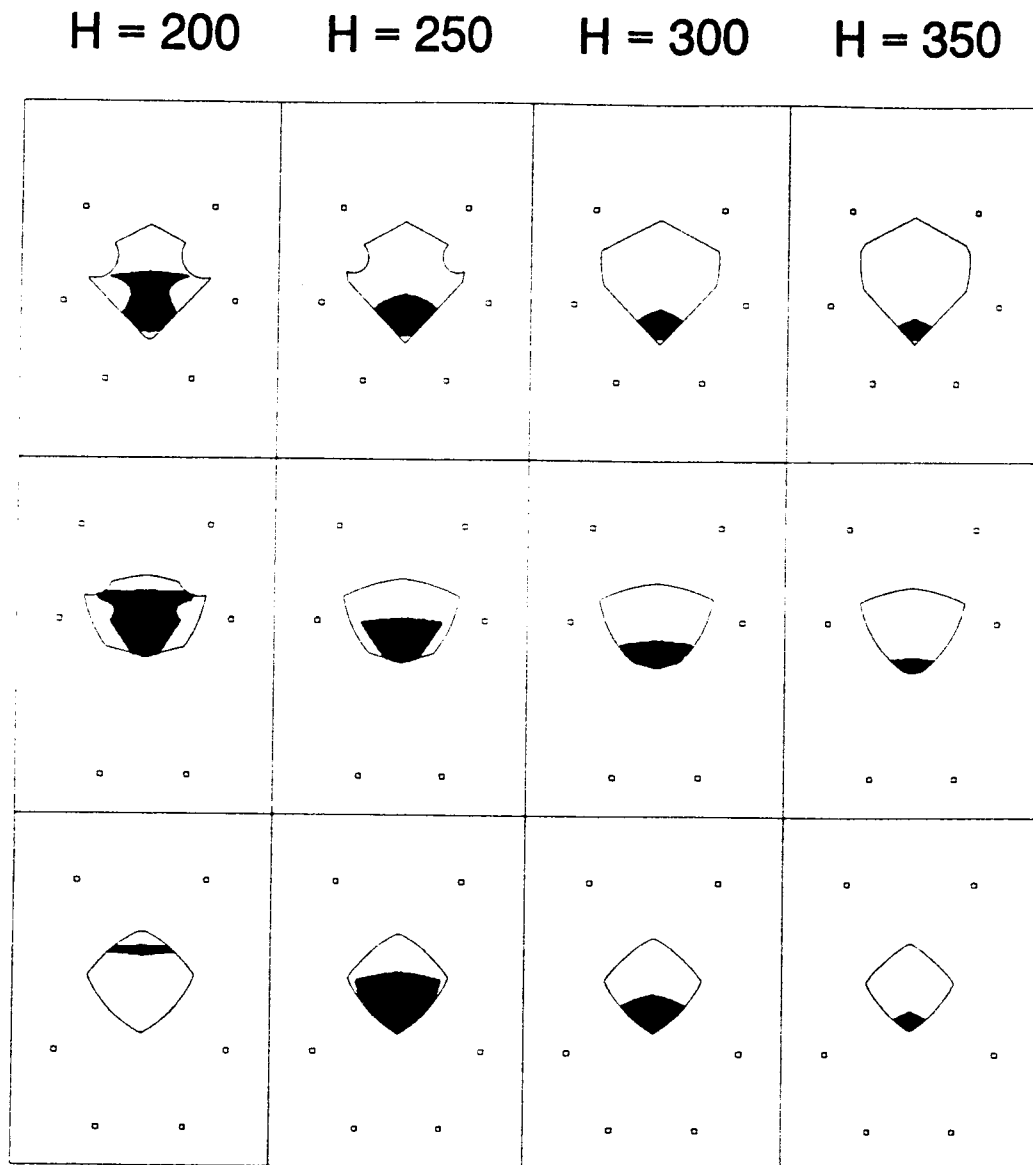


Figure 4.12 : Body workspace of machine at pitch of -5 degrees, with variation in height of mass center of table and foot position, and front body coxa joints below by 50 mm.

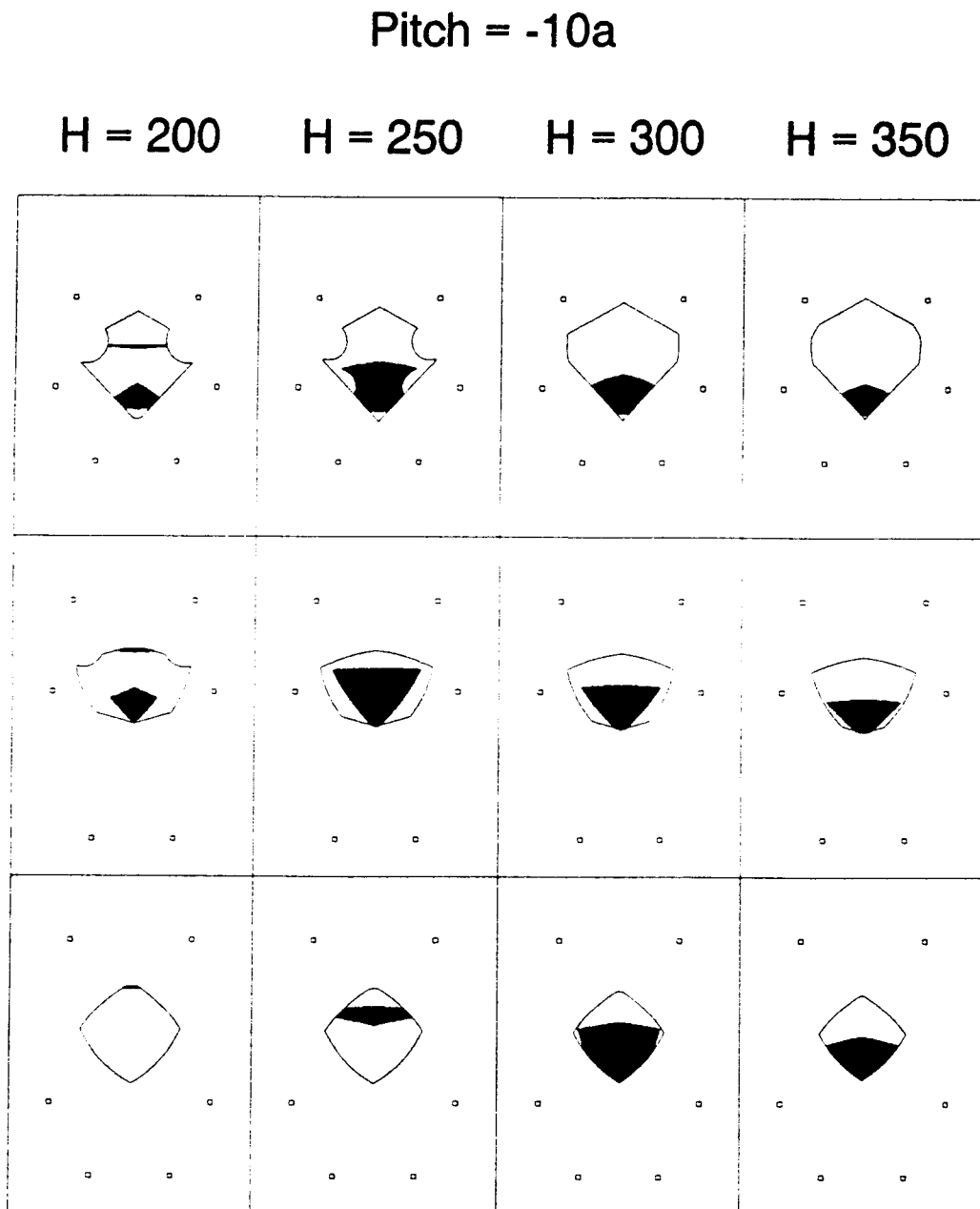


Figure 4.13 : Body workspace of machine at pitch of -10° degrees, with variation in height of mass center of table and foot position, and front body coxa joints below by 50 mm.

Pitch = -15a

H = 200

H = 250

H = 300

H = 350

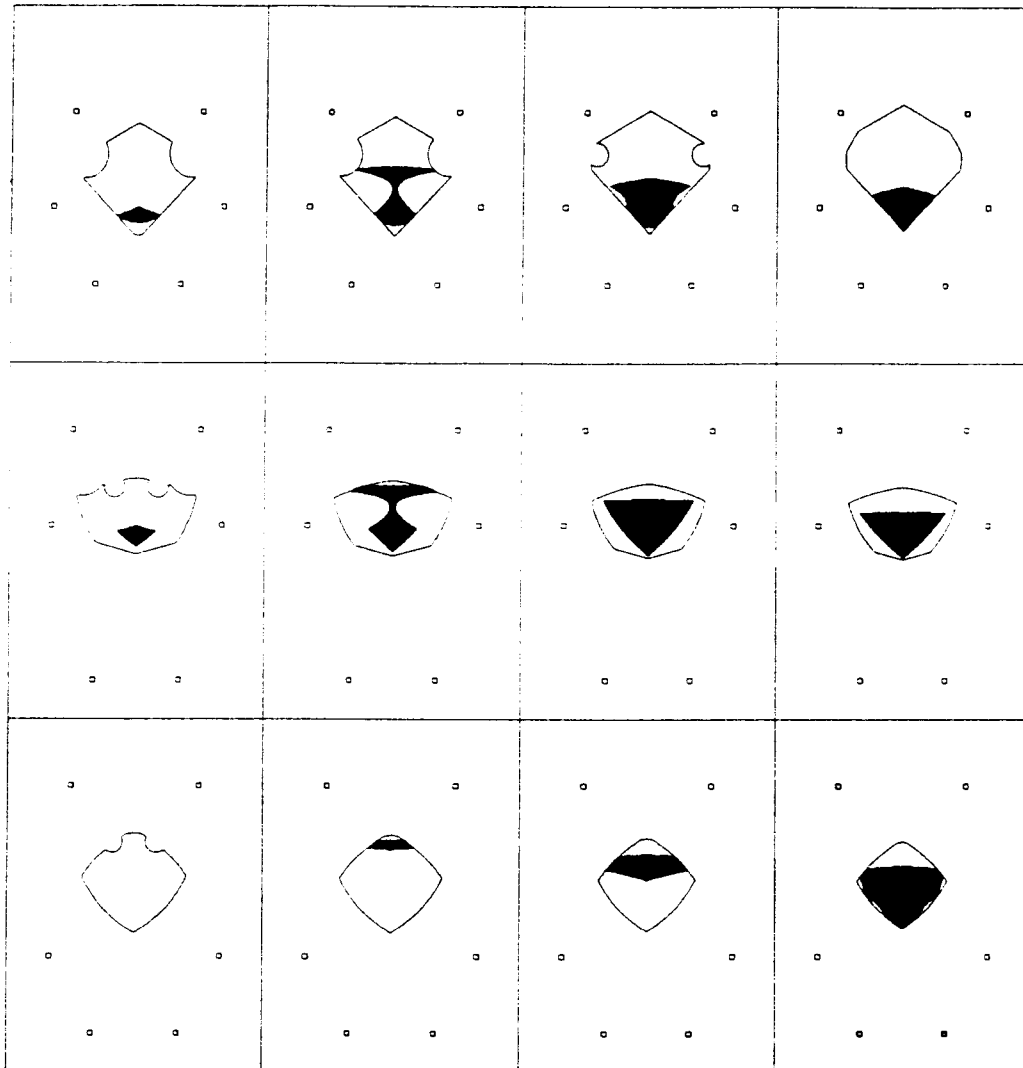


Figure 4.14 : Body workspace of machine at pitch of -15 degrees, with variation in height of mass center of table and foot position, and front body coxa joints below by 50 mm.

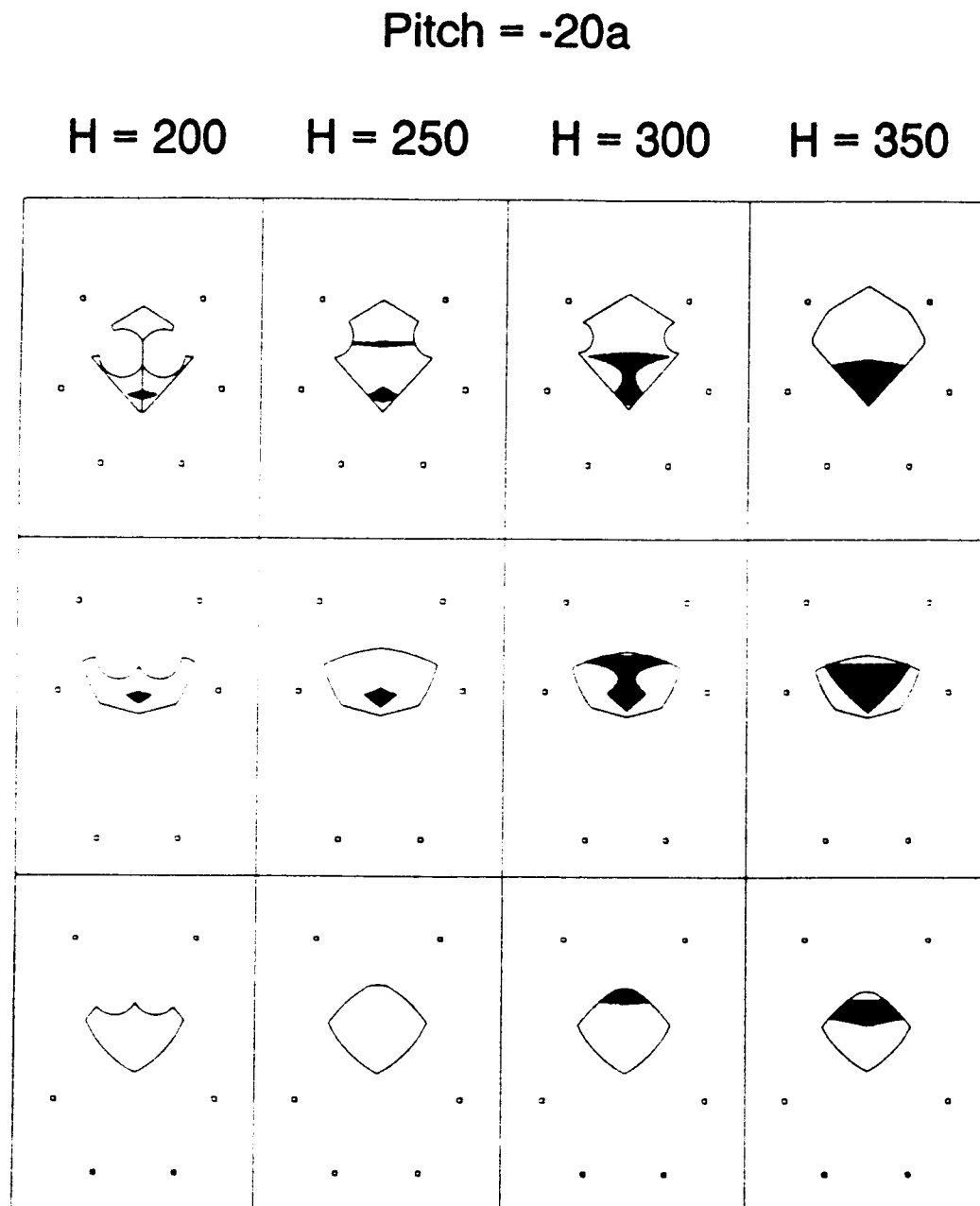


Figure 4.15 : Body workspace of machine at pitch of -20 degrees, with variation in height of mass center of table and foot position, and front body coxa joints below by 50 mm.

From the above plots, most of the conclusions that can be drawn are same as that of the conclusions with all the body coxa joints at the same height. In addition to those conclusions, following conclusions can be drawn :

- (i) It was found that with same pitch, foot position and height of the mass center of the machine the design having body coxa joints at the same height have smaller body workspace as compared to the design in which the front body coxa joints are dropped down by 50 mm.
- (ii) It can also be seen from the figures that, for same foot position and height of center of mass of machine, decreasing the height of front body-coxa joints by 50 mm, the body workspace is similar to the body workspace with 10 degrees more pitch and all the body coxa joints on the same plane.

It was also found during the study, that it is important to determine magnitude of forces as the body is moved along y_g axis. The plot of forces, as the center of mass of machine is moved along y_g axis gives a range of motion for moving forward in a straight line. The procedure followed for finding leg forces that the body coxa joints exert on body was similar to the procedure for finding kinematic workspace and body workspace, the only difference being that the grid search is not done along x_g axis. Center of mass of the body is only moved along y_g axis with step size of 10 mm.

Figure 4.16 shows the plot of forces in the right legs as mass center of machine is moved along y_g axis. The units are consistent with the units used in system description file, that is units of forces will be gm.mm/s^2 . The forces thus calculated if divided by 10^6 gives force in Newtons. Figure 4.16 shows the variation of magnitude of forces in right legs (the forces in left legs will have same magnitude as that of right legs) as the body is moved along y_g axis. In the figure positive forces represent compression and negative forces represent tension. The useful length will be the length in which all the forces are positive, that is the body can be made to stand within that length. Figure 4.16 shows plot of forces with pitch of -15 degrees, foot position 12 and height of center of mass of machine equal to 250 mm. It can be seen in figure that the range in which all the three forces are positive is about 160 mm. Doing these plots also gives a good idea of the body workspace.

It can be seen in Figure 4.16, that force in front leg crosses the zero line at y_g coordinate of center of mass equal to -40, at which point all the legs become compressive. At all the positions before this point the force in front leg was tensile, and thus the positions were outside the force workspace. It can also be seen that at y_g coordinate of mass center of machine equal to 120, the force in the middle leg crosses the zero line and becomes tensile, thus making all the positions after this point outside the force workspace. It can also be seen that the force in hind leg always

remain compressive. Thus between the y_g coordinate of mass center of machine from -40 to 120 the forces in all the legs were compressive and thus this range constitute the force workspace along y_g axis.

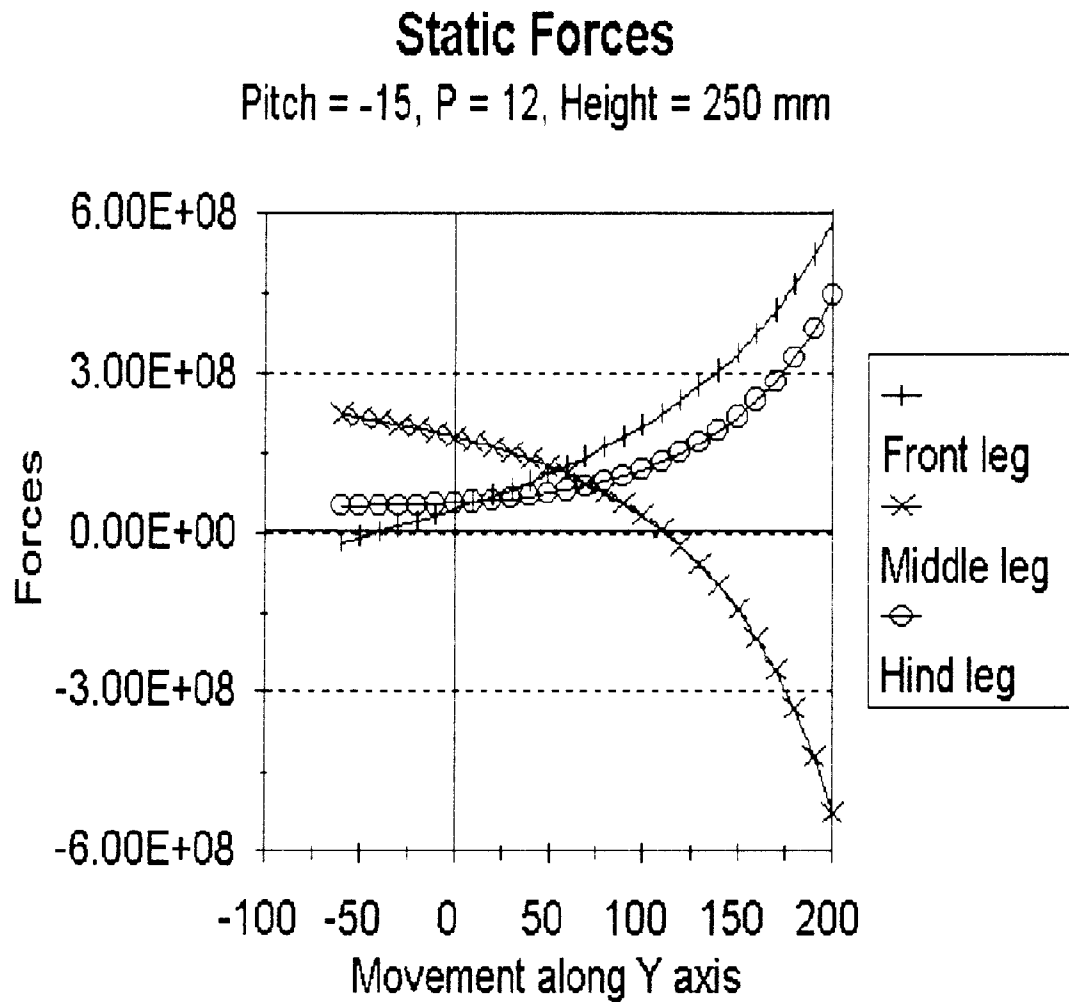


Figure 4.16 : Variation of Forces(gm.mm/s²), applied by legs on body, at the body-coxa joints as the body is moved along y_g axis.

Following tables show the length along y_g axis in which all the legs are in compression as the body is moved along y_g axis. The tables show the variation of the length with change in pitch, body height and foot position. In the tables the variation in foot position is done along the columns and along the rows the height of the body is varied. Table 4.4, 4.5, 4.6, 4.7 and 4.8 shows the variation in the length along y_g axis with body pitch of 0, -5, -10, -15 and -20 degrees.

As seen earlier there is substantial increase in body workspace of the machine as the front body coxa joints are moved down by 50 mm. Table 4.9, 4.10, 4.11, 4.12 and 4.13 shows variation in the length along y_g axis with pitch of 0, -5, -10, -15 and -20 and with front body coxa joint moved down by 50 mm.

Table 4.4 : Variation of length along y_g axis where the machine can stand.
Pitch = 0 (degrees).

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	20	20	20	20
P = 12	0	0	0	0
P = 19	0	0	0	0

Table 4.5 : Variation of length along y_g axis where the machine can stand.
Pitch = -5 (degrees).

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	60	30	30	20
P = 12	60	0	0	0
P = 19	70	0	0	0

Table 4.6 : Variation of length along y_g axis where the machine can stand.
Pitch = -10 (degrees).

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	160	90	60	40
P = 12	160	60	0	0
P = 19	290	170	80	0

Table 4.7 : Variation of length along y_g axis where the machine can stand.
Pitch = -15 (degrees).

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	230	160	110	90
P = 12	240	160	90	50
P = 19	160	290	210	130

Table 4.8 : Variation of length along y_g axis where the machine can stand.
Pitch = -20 (degrees).

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	210	200	160	130
P = 12	240	210	150	100
P = 19	50	210	300	230

Table 4.9 : Variation of length along y_g axis where the machine can stand.
Pitch = 0 (degrees), and front body-coxa joint moved down by 50 mm.

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	220	120	80	60
P = 12	200	70	0	0
P = 19	290	140	40	0

Table 4.10 : Variation of length along y_g axis where the machine can stand.
Pitch = -5 (degrees), and front body-coxa joint moved down by 50 mm.

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	310	200	140	100
P = 12	300	210	140	80
P = 19	50	290	190	100

Table 4.11 : Variation of length along y_g axis where the machine can stand.
Pitch = -10 (degrees), and front body-coxa joint moved down by 50 mm.

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	200	260	190	140
P = 12	200	280	220	170
P = 19	10	100	290	210

Table 4.12 : Variation of length along y_g axis where the machine can stand.
Pitch = -15 (degrees), and front body-coxa joint moved down by 50 mm.

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	110	290	240	190
P = 12	140	330	270	220
P = 19	0	50	130	290

Table 4.13 : Variation of length along y_g axis where the machine can stand.
Pitch = -20 (degrees), and front body-coxa joint moved down by 50 mm.

	H = 200 mm	H = 250 mm	H = 300 mm	H = 350 mm
P = 10	100	150	250	220
P = 12	110	150	280	250
P = 19	0	0	70	140

From above tables following conclusions can be drawn :

- (i) Increase in the height of mass center of machine decreases the length along y_g axis, where all the leg forces are in compression, that is the forces exerted by the legs on the body are positive.
- (ii) Increase in the pitch of machine increases the length along y_g , where all the forces are in compression.

(iii) For same foot position, height of center of mass of machine and pitch, making height of front body-coxa joints down by 50 mm increases the length along y_0 , where all leg forces are compressive.

(iv) In some cases it was also seen that with foot position 12 and 19, increase in length where all the forces are compressive is possible, if there is

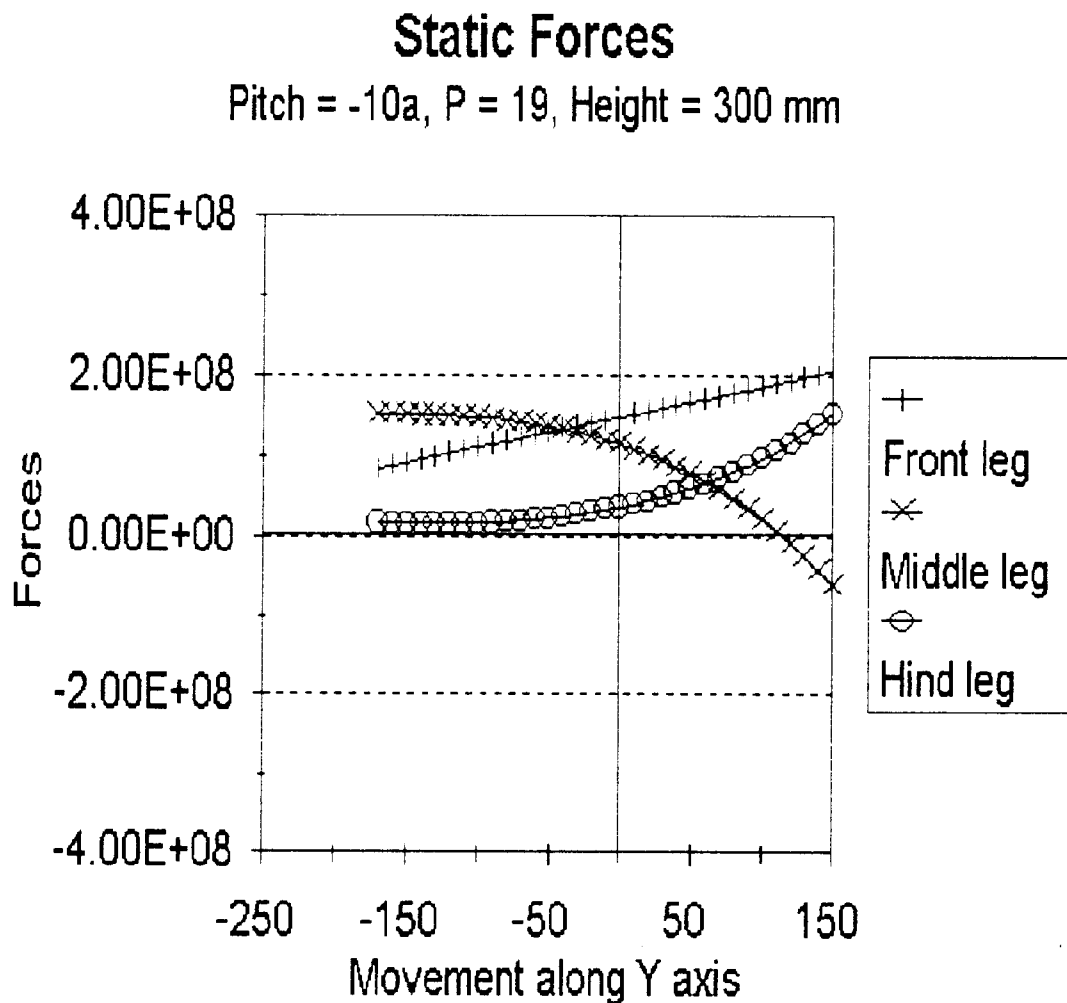


Figure 4.17 : Variation of Forces(gm.mm/s^2), applied by legs on body, at body-coxa joints as body is moved along y_0 axis.

increase in kinematic workspace. Figure 4.17 shows one of the cases where the compressive force length is limited by kinematic workspace length. As explained earlier the search for force length was started at -400 mm in y_0 and the machine was moved along positive y_0 axis, at y_0 equal to -170 kinematic workspace is the limiting factor for the force length.

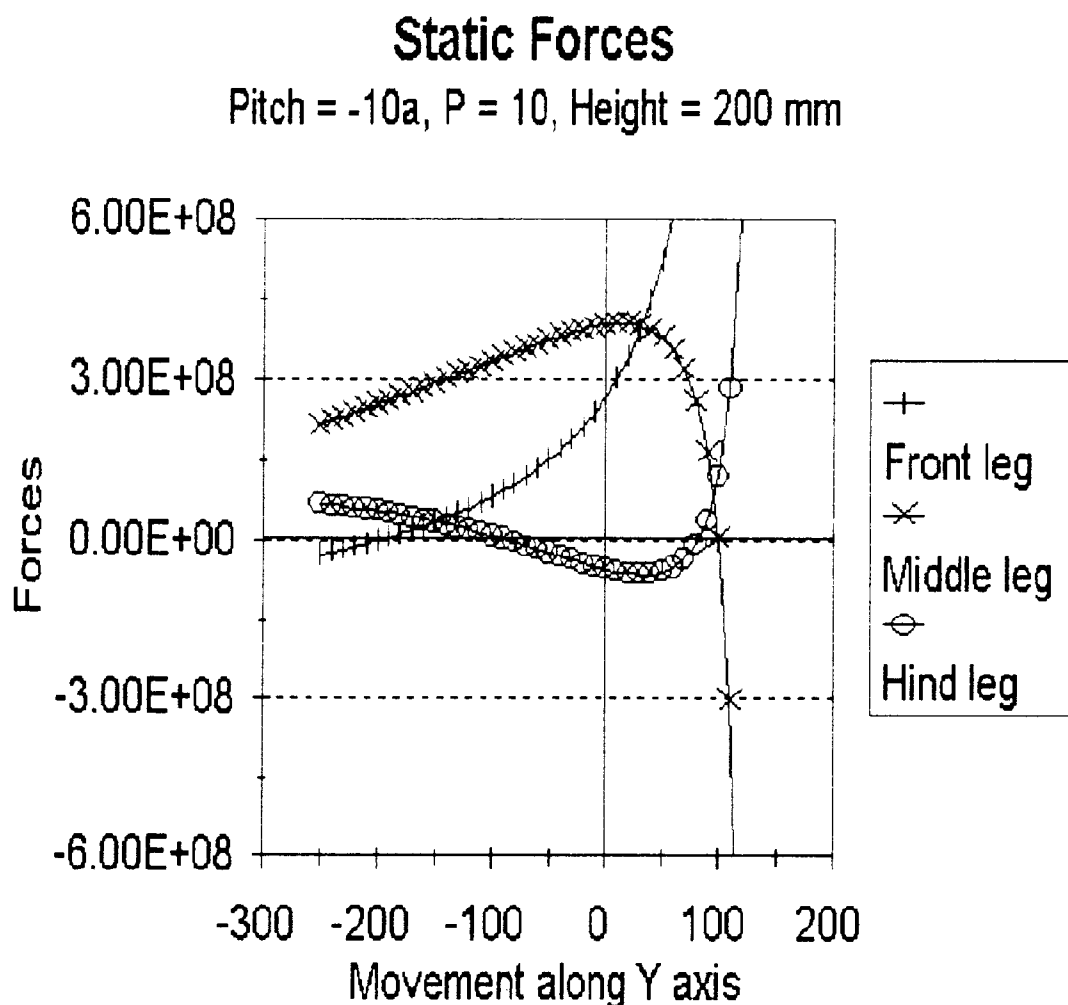


Figure 4.18 : Variation of Forces(gm.mm/s²), applied by legs on body, at body-coxa joints as body is moved along y_0 axis.

On plotting the variation of forces with position of mass center of machine along y_g axis, it was found that in most of the plots that forces in front and middle legs were the limiting forces for the length of force workspace along y_g axis. It was also seen in some of the cases that the force workspace within the kinematic workspace of the machine was split in two parts (refer Figure 4.18). It can be seen in Figure 4.18 that the split in the force workspace occurs because of the force in the hind leg crosses the zero line and becomes tensile. Later the force in the hind leg again becomes compressive, thus giving the places in which all the legs are in compression.

Later the study of the variation of the force along y_g , was done using Analytix. Figure 4.19 shows the projection of leg forces on the yz_g plane. The projection of forces in yz_g plane is done since the whole body of machine is symmetrical about yz_g plane. The X component of forces in right legs cancel the X component of forces in left legs, also the moment caused due to X component of forces in right legs cancel the moment caused due to X component of forces in left legs. In Figure 4.19, A is the point of intersection of the forces in the front and hind leg, B the point of intersection between forces in middle and hind leg and C the point of intersection between forces in front and middle leg. In the figure COM is the location of center of mass of machine, F_h , F_m and F_f are the location of hind, middle and front foot respectively, Ch , Cm and Cf are the location of hind, middle and front body-coxa joints respectively. In Figure 4.19 point B

directly below the mass center of machine, and thus the force in the middle leg should be zero. It was seen that in most cases the point A or B comes directly under the center of mass of the machine. But in the cases where the body workspace was split point C comes directly under center of mass of machine thus making the force in the hind leg equal to zero. Similar analysis can be done with variation in foot positions, height of center of

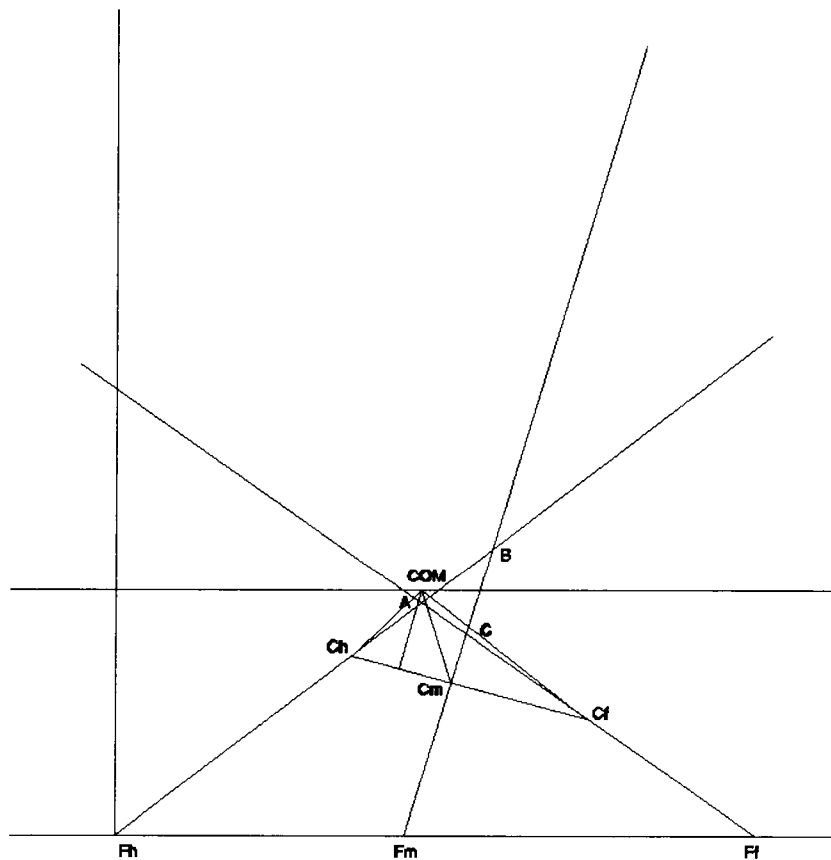


Figure 4.20 : Projection of leg forces on yz_g plane.

mass of machine and pitch of body. The limitation of using this model occurs, when there is asymmetry in machine, which could be due to foot positions, roll or yaw.

5. Screw Axis

It is known that any general three dimensional motion, regardless of how the motion occurs, can be described as a screw motion, a combination of a rotation about a screw axis and a translation along the screw axis (Ball, 1900 and Hunt, 1978). Such a motion is called a screw motion and in kinematics it is referred to as Chasle's theorem. Figure 5.1 shows the screw axis for spatial motion of a rigid body.

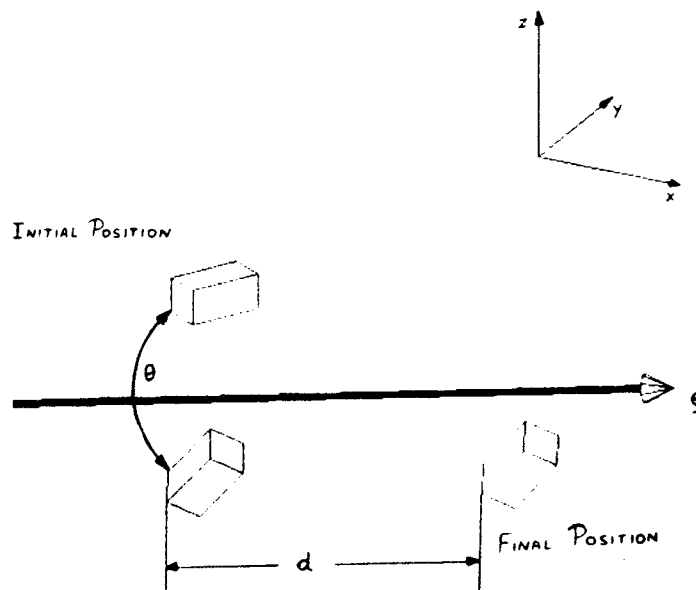


Figure 5.1 : Description of screw axis.

Thus the spatial motion of a rigid body can be completely specified by a set of parameters defining the screw axis, magnitude of translation along the screw axis and magnitude of rotation about the screw axis. It is also known that position and orientation of a rigid body is completely defined, if the coordinates of three non collinear points on the body are known. Thus if initial and final position coordinates of three non collinear points on a rigid body are known then initial and final position and orientation of rigid body are also known. Thus motion of a rigid body can be described by the initial and final position coordinates of three non collinear points on the body. Therefore, the screw parameters can be determined if the initial and final position coordinates of three non collinear points are known. The problem of determining the screw parameters from the set of initial and final position coordinates of three non collinear points is that of an inverse kinematics.

The problem of determining screw parameters from initial and final position has resulted in development of number of algorithms. Angel's method (Angeles, 1986), Laub and Shiflett's method (Laub et al 1982) and Rodrigue's formula (Bottema et al, 1979), are some of the algorithms developed for determining screw parameters. These algorithms yield equivalent results even though they are based on different concepts. It is important that these methods are efficient, accurate and sensitive to data errors. Fenton et al, 1990, compared five algorithms for calculating screw parameters.

The screw parameter can be divided in two groups. The first group of parameters define the position and orientation of screw axis and the second group specifies the magnitude of rotation (θ_i) about screw axis and magnitude of translation along screw axis (d_i). For some of the methods the second group contains the pitch of the screw, which is the ratio of magnitude of translation along screw axis and magnitude of rotation about screw axis (d_i/θ_i).

As discussed by Plücker (Hunt ,1978), any line can be uniquely determined by its Plücker coordinates, that is, it can be defined by its three direction cosines expressed by a unit vector (\mathbf{e}) with a position vector \mathbf{A} locating point A on the line. Thus to specify screw motion eight scalar parameters are needed; three components of \mathbf{e} , three components of \mathbf{A} and magnitudes of θ_i and d_i .

However components of \mathbf{e} must satisfy the constraint

$$e_x^2 + e_y^2 + e_z^2 = 1 \quad \text{.. 5.1}$$

Point A can be chosen such that the vector \mathbf{A} is perpendicular to the screw axis. In this case a second constraint is

$$\mathbf{e}^T \mathbf{A} = 0 \quad \text{..5.2}$$

Therefore, of the eight scalar screw parameters only six are independent, and the motion can be defined by determining these six independent screw parameters from the initial and final position coordinates of three non collinear points of the rigid body.

Spatial displacement can also be conveniently described by a 4x4 transformation matrix $[T]$, so that

$$[P_f] = [T] [P_i] \quad \text{..5.3}$$

where $[P_i]$ is a 4x3 matrix of the homogenous coordinates of the initial position of the three points, $[P_f]$ is a 4x3 matrix of the homogenous coordinates of the final position of the three points, and

$$[T] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} [R] & P \\ 0 & 1 \end{bmatrix}$$

where $[R]$ is the 3x3 rotation matrix and P is the translation vector.

Matrix [T] contains 12 scalar elements which are to be determined, but the properties of transformation provide the following six constraints :

$$r_j^T r_j = 1 \quad \text{for } j = 1, 2, 3 \quad \text{..5.5}$$

$$r_i^T r_j = 0 \quad \text{for } i = 1, 2, 3 \text{ and } i = 1, 2, 3 \text{ and } i \neq j$$

..5.6

where r_i and r_j are column vectors of [R]. Therefore it is clear that the number of independent variables in [T] is also six. Upon obtaining the transformation [T], the required screw parameters can be alternatively defined as determining the six independent elements in the transformation [T] from the initial and final position coordinates of three non collinear points on a rigid body. Also if screw parameters are known then transformation matrix [T] can be found and if transformation matrix is known the screw parameters can be found.

5.1 Rodrigue's Formula (Bottema and Roth method, Bottema et al 1979) :

This algorithm does not need to form the rotation matrix [R], instead it can directly provide the solution of screw parameters. The main disadvantage of this method is that, if data error exists, three different solutions for the screw parameters may be obtained depending on the

different order in which the points are considered. However, Fenton et al 1990, showed that the difference between the solutions is insignificant compared to the results obtained by other methods when using inexact data.

Based on vector manipulation, the magnitude of rotation (θ) and the unit vector \mathbf{e} are obtained by using equation 5.7 to 5.9.

$$P1 = [(\mathbf{p}_{f3}-\mathbf{p}_{f2})-(\mathbf{p}_{i3}-\mathbf{p}_{i2})] \times [(\mathbf{p}_{f1}-\mathbf{p}_{f2})-(\mathbf{p}_{i1}-\mathbf{p}_{i2})] \quad ..5.7$$

$$P2 = [(\mathbf{p}_{f3}-\mathbf{p}_{f2})-(\mathbf{p}_{i3}-\mathbf{p}_{i2})] \cdot [(\mathbf{p}_{f1}-\mathbf{p}_{f2}) + (\mathbf{p}_{i1}-\mathbf{p}_{i2})] \quad ..5.8$$

$$\tan(\theta/2)\mathbf{e} = P1/P2 \quad ..5.9$$

In the above equation \mathbf{p}_{i1} , \mathbf{p}_{i2} and \mathbf{p}_{i3} are the initial position of points \mathbf{p}_1 , \mathbf{p}_2 and \mathbf{p}_3 respectively, on a rigid body, while \mathbf{p}_{f1} , \mathbf{p}_{f2} and \mathbf{p}_{f3} are the final position of the same three points. The position vector of point A lying on screw axis can be determined from :

$$\begin{aligned} \mathbf{A} = (1/2)[\mathbf{p}_{i1} + \mathbf{p}_{f1} + ((\mathbf{e}) \times (\mathbf{p}_{f1}-\mathbf{p}_{i1})/\tan(\theta/2)) \\ - \mathbf{e} \cdot (\mathbf{p}_{f1} + \mathbf{p}_{i1})\mathbf{e}] \quad ..5.10 \end{aligned}$$

and finally magnitude of translation can be computed from :

$$d = \mathbf{e} \cdot (\mathbf{p}_{f1} - \mathbf{p}_{i1}) \quad \text{..5.11}$$

which is projection of change in coordinate of any one of the three points, on unit vector along screw axis.

When using Bottema and Roth method, it should be kept in mind that order of the three points cannot be always chosen arbitrarily. In case when rigid body has pure rotation about a line joining any two of three points, if the two points are chosen to be as \mathbf{p}_2 and \mathbf{p}_3 , then the term $(\mathbf{p}_{f3} - \mathbf{p}_{f2}) - (\mathbf{p}_{i3} - \mathbf{p}_{i2})$ in the denominator of equation 5.8 vanishes, and the equation becomes undefined.

5.2 Falling Screw of Machine

After having found the body workspace of machine, that is the position of center of mass of machine where it can be made to stand, the next step is to make the body move. To make body move one leg, has to be picked up and placed, in turn, at some other position. As explained earlier the machine has six degrees of freedom, one coming from each leg. For making the machine stand and be in static equilibrium the constraints are applied on each leg, that is torques are applied at femur-tibia joints. For

lifting one leg, constraint on that leg is removed and machine gets one degree of freedom.

Since there are number of forces acting on the machine, which include the gravitational forces and forces exerted by legs on machine, the machine tends to fall. This falling motion can be represented by a screw. For finding the falling screw of machine, the body workspace of machine was used to find the position where the force in one leg is nearly zero. These positions of center of mass of machine, where the force in one of the legs is zero, are important since lifting that leg should result in movement of machine, but at that position a small disturbance of machine will make the machine move and fall. Also it was assumed for the study that there is no friction at the joints. It was also seen that the force in hind leg rarely crosses zero (except split body workspace), thus the hind leg was lifted at the position where the force in the leg was low. The falling motion of the machine was studied at two positions in y_g axis, that is on the two limiting ends of the force workspace. As a result it was decided to pick up the leg and study the falling motion of machine at position near the points where the force in the leg is zero.

To find the screw axis of machine, as one leg is lifted, dynamic simulation of machine was done using the code generated by SD/FAST. To simulate the picking of leg, the prescribed motion on that leg was turned off

and the prescribed motion on rest of the legs were turned on. Code was written in C to find the screw axis of machine. As discussed earlier coordinates of three points on the body are needed to completely specify the position and orientation of a body, three points on machine were arbitrarily chosen. The final position coordinates of these three points were found, thus giving the data needed for finding screw parameters.

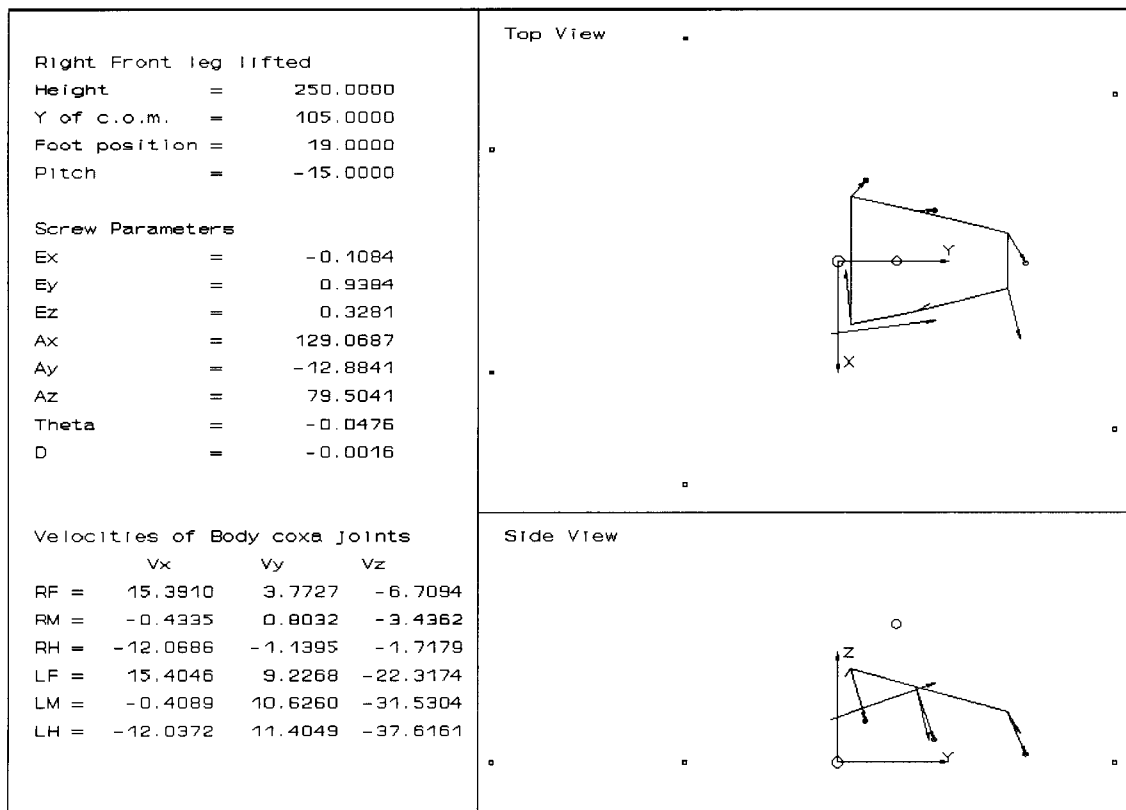


Figure 5.2 : Falling screw of machine.

After having found the screw parameters code in AutoLisp was written to plot the motion of machine in AutoCAD. Figure 5.2 shows the falling screw of the machine at a particular set of parameters, that is for the figure the body pitch is -15 degrees, foot position is 19 and height of center of mass of body is 250 mm. The plot shows the velocities of body coxa

joints and the screw axis, after lifting right front leg of the machine. In the figure E_x , E_y , E_z represent the unit vector of the screw axis; A_x , A_y , A_z represent the perpendicular to the screw axis from the origin of ground coordinate system; θ and D represent the magnitude of rotation along screw axis and translation about screw axis respectively. In Figure 5.2 the arrow across the body shows the screw axis of machine. Figure 5.2 shows the top and side view of the machine along with the direction of the velocity of the body-coxa joints. The square boxes in the figure represent the foot position. The plane trapezoid shown in the figure connects the body coxa joints of machine. The circle above the body coxa joints represents the position of center of mass of machine in ground coordinate system.

The main purpose of doing this analysis, is to see if the falling motion of machine can be utilized to make the body move along positive y_g axis. To make the body move along y_g axis, one or more of the following should be seen in screw parameters:

Case I : The screw axis should be parallel to y_g axis, and the pitch of the screw should be infinity. For this case the position of point A is arbitrary, that is it does not affect the motion of the body. Thus for the motion of machine along y_g axis the E_y should be large and pitch should be large.

Case II : The screw axis should be parallel to x_g axis, that is E_x should be large; the pitch of the screw should be zero and A_z should be as large as possible. As a result of this combination, the body tends to follow the motion about the screw axis along a circle, which tends to be straight line as the screw axis moves away from the body.

It was seen in the discussion of body workspace of machine that :

- (i) Decreasing pitch of machine by 5 degrees and simultaneously increasing the height by 50 mm gives similar body workspace.
- (ii) Increasing pitch of machine by 10 degrees and lowering body coxa joints by 50 mm gives similar body workspace.

Table 5.1 : Machine parameters having similar body workspace.

Pitch	Height	Foot position
-10	200 mm	10,12,19
-15	250 mm	10,12,19
-20	300 mm	10,12,19
0a	200 mm	10,12,19
-5a	250 mm	10,12,19
-10a	300 mm	10,12,19
-15a	350 mm	10,12,19

Thus to see the variation of screw parameters, the machine parameters having similar body workspace were selected. Table 5.1 shows the combination of parameters which were selected for doing the analysis on falling screw of machine. Screw axis was found near the positions where the force in one leg is nearly zero. Figure 5.3 shows the variation of E_x or X component of unit vector of screw axis; Figure 5.4 shows the variation of E_y or Y component of unit vector of screw axis; Figure 5.5 shows the variation of E_z or Z component of unit vector of screw axis; Figure 5.6 shows the variation of A_x or X component of perpendicular to unit vector of screw axis; Figure 5.7 shows the variation of A_y or Y component of perpendicular to unit vector of screw axis; Figure 5.8 shows the variation of A_z or Z component of perpendicular to unit vector of screw axis; Figure 5.9 shows variation of pitch of falling screw of machine. In the figures, the curves shows the screw axis parameters at upper limit and lower limit of force length with both the designs, that is, with all body-coxa joints in the same plane and another with front body coxa joints dropped down by 50 mm. In the figures the rectangle symbol is for the design having body-coxa joints below by 50 mm and the ellipse symbol is for design having body-coxa joints in the same plane. Also the curve connecting shaded symbols are at upper limit of force length, that is the curves connecting the shaded rectangles are for front body coxa joints down by 50 mm and are the upper limit of force length.

Values of X component of unit vector of screw axis

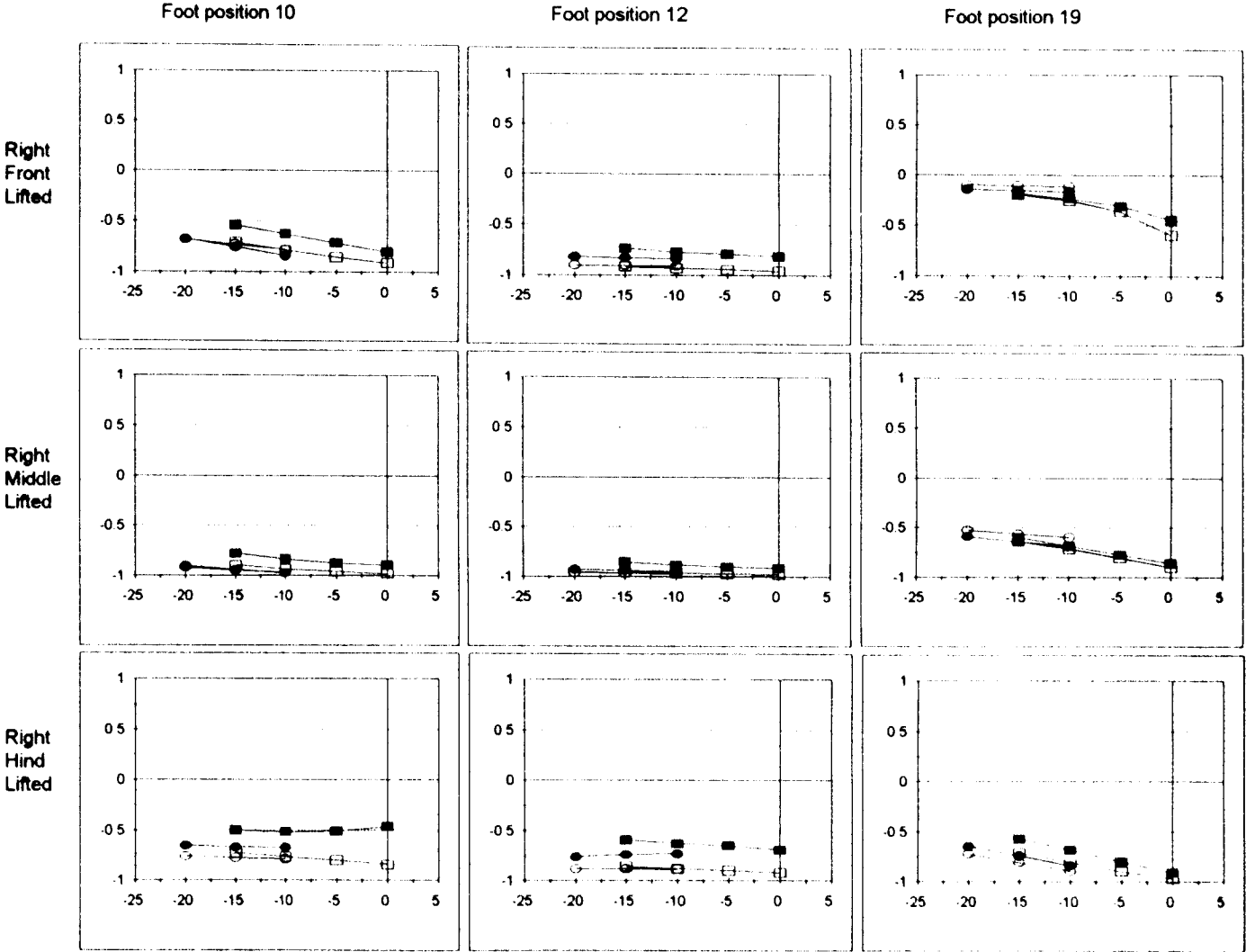


Figure 5.3 : Variation of Ex or X component of unit vector of screw axis.

Values of Y component of unit vector of screw axis

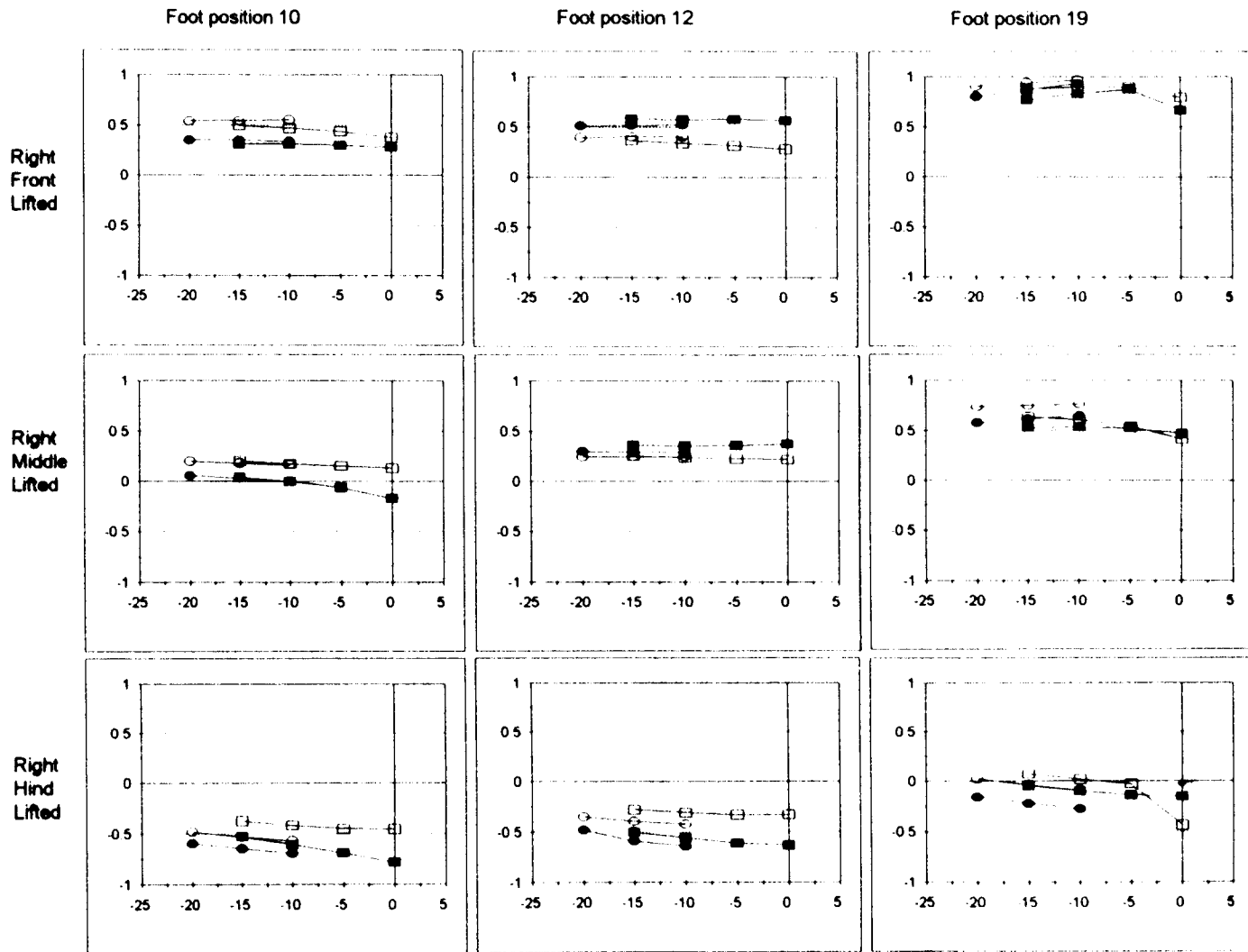


Figure 5.4 : Variation of E_y or Y component of unit vector of screw axis.

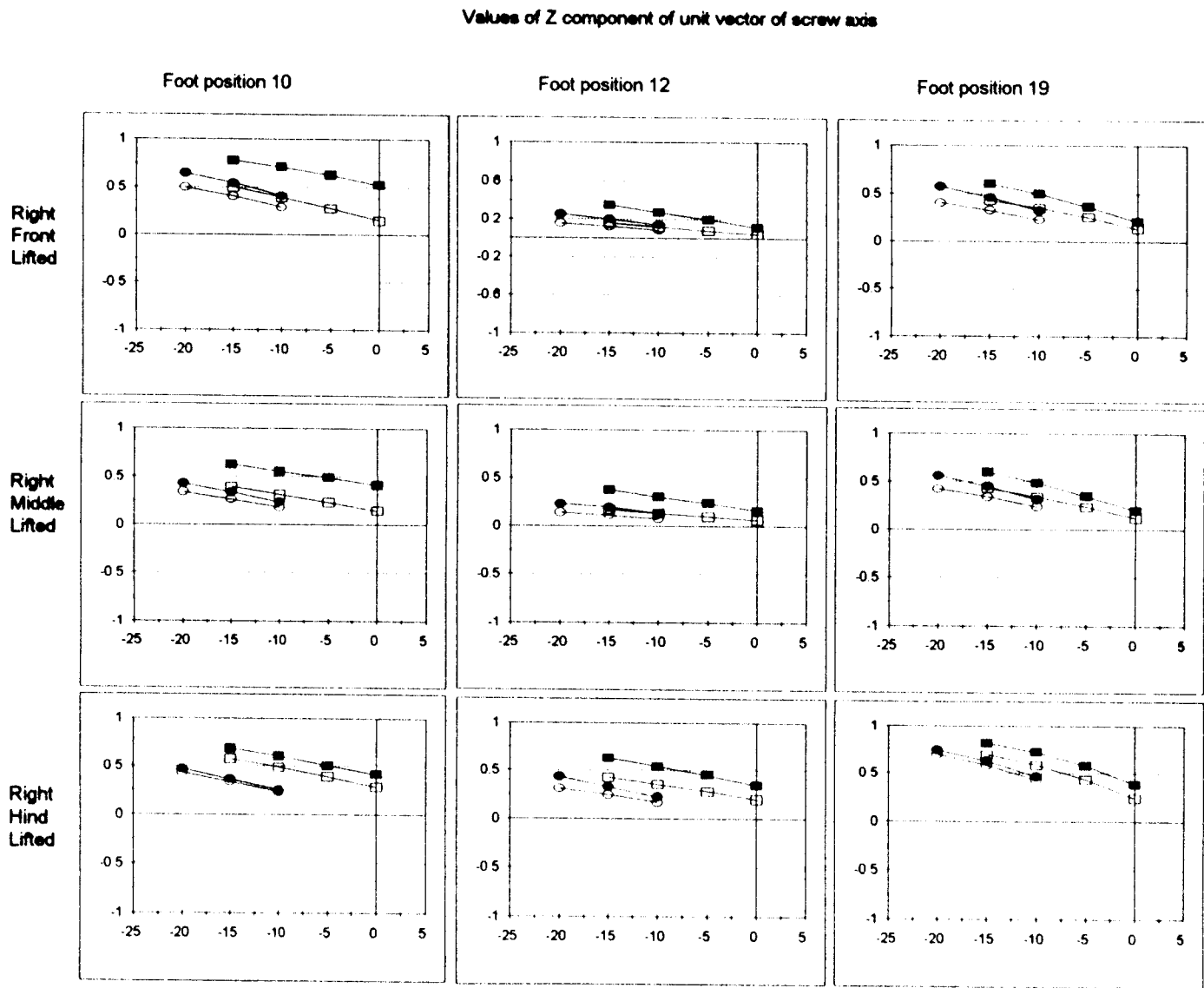


Figure 5.5 : Variation of E_z or Z component of unit vector of screw axis.

Values of X component of perpendicular to unit vector of screw axis

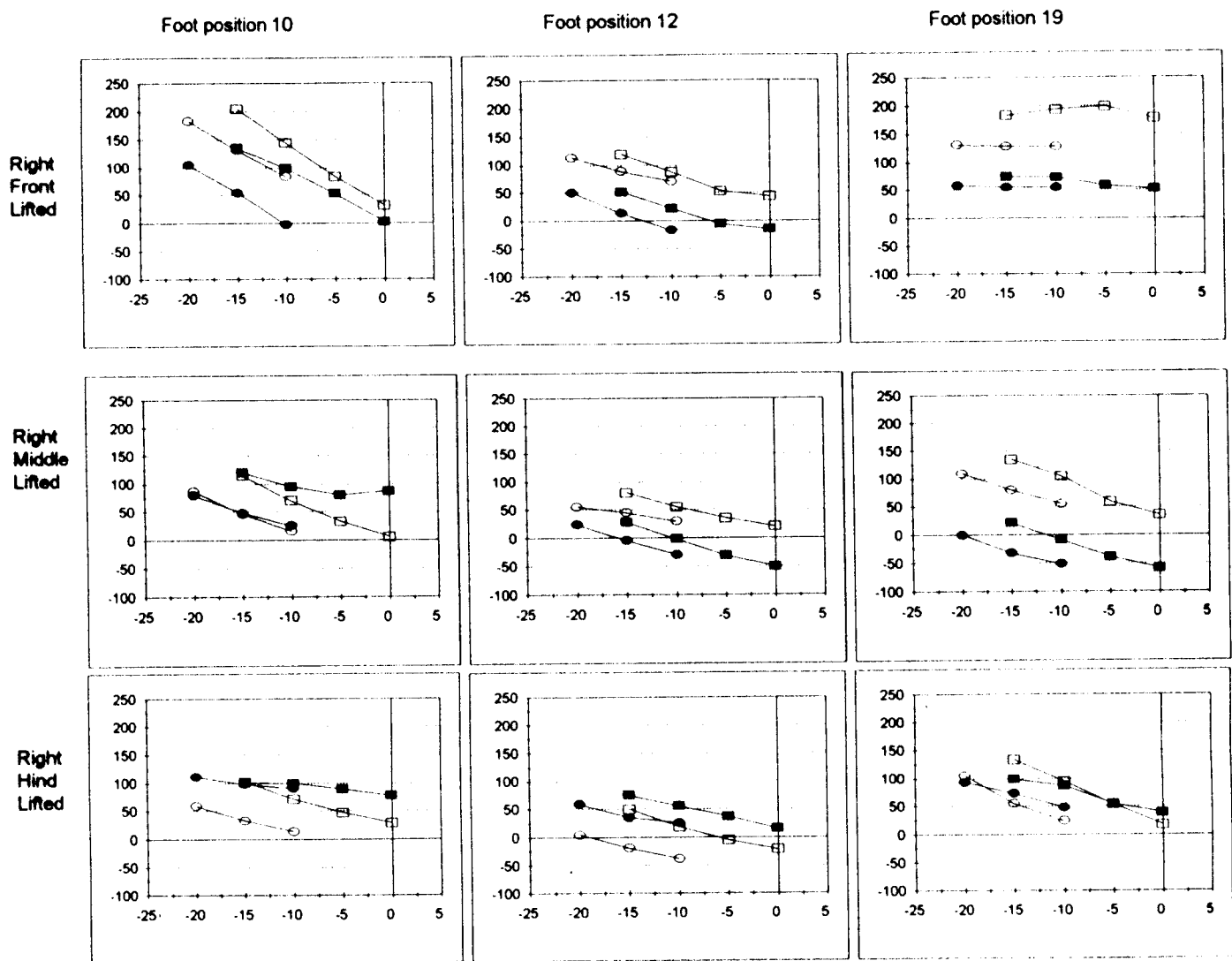


Figure 5.6 : Variation of A_x or X component of perpendicular to unit vector of screw axis.

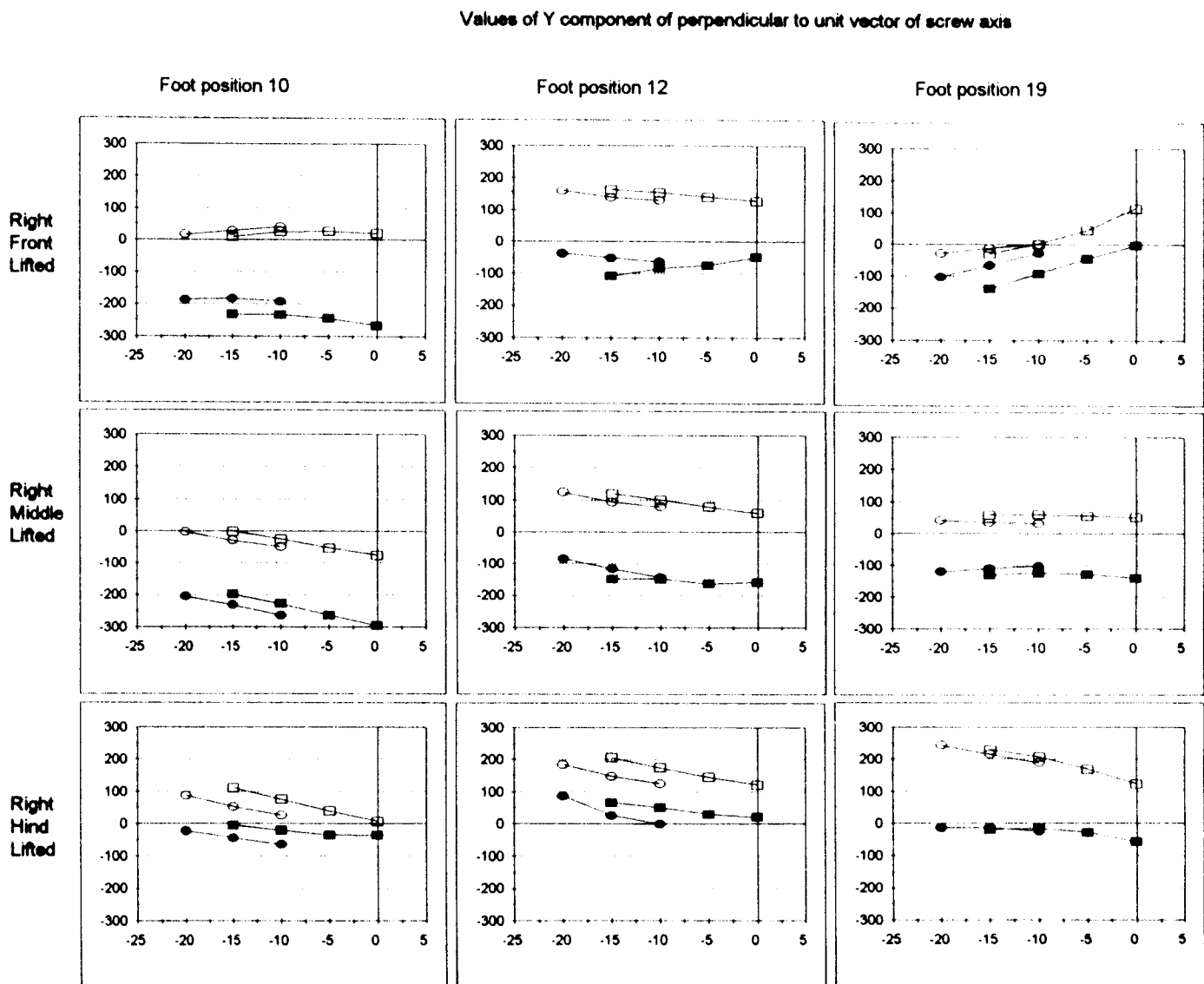


Figure 5.7 : Variation of A_y or Y component of perpendicular to unit vector of screw axis.

Values of Z component of perpendicular to unit vector of screw axis

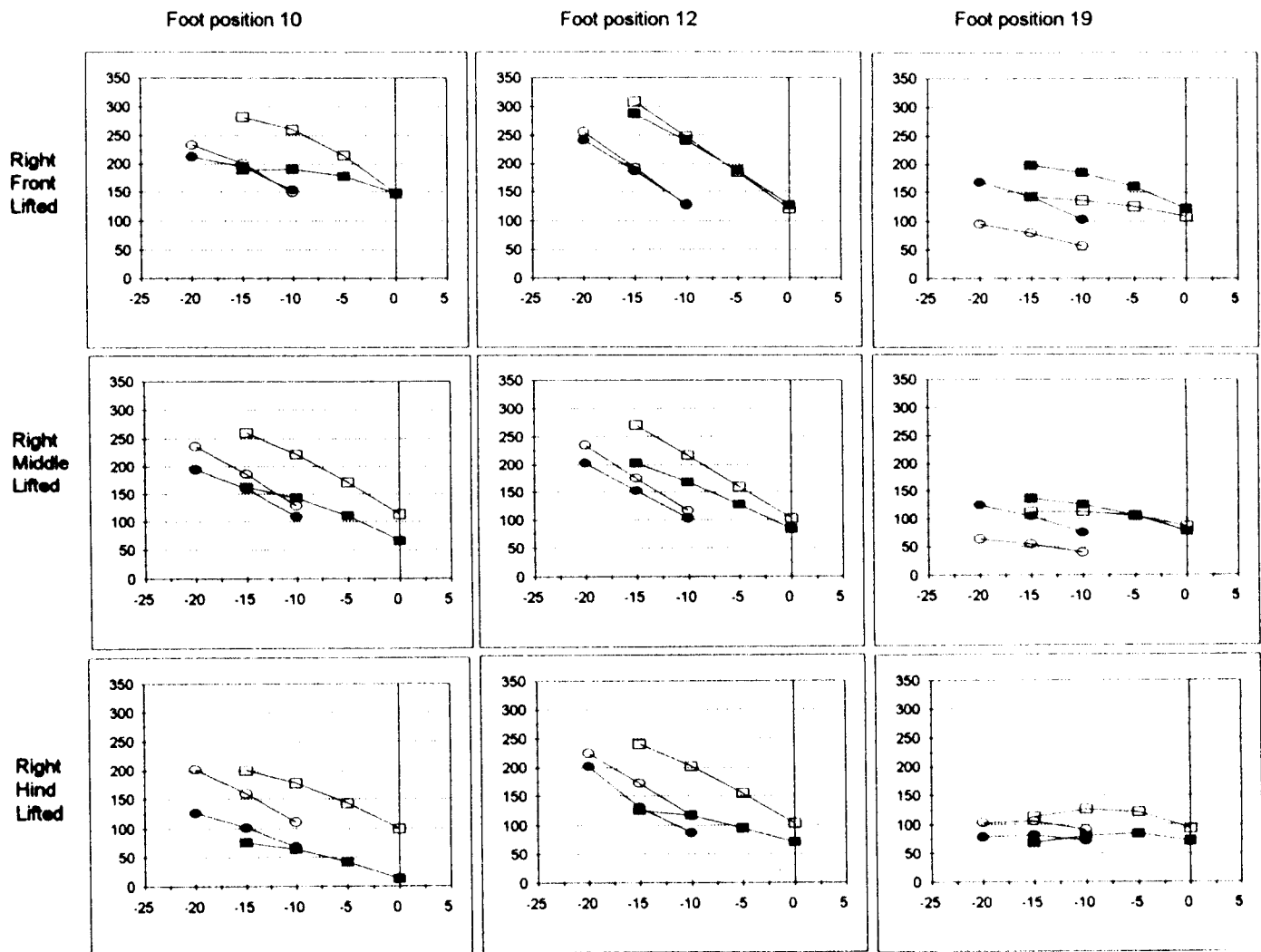


Figure 5.8 : Variation of A_z or Z component of perpendicular to unit vector of screw axis.

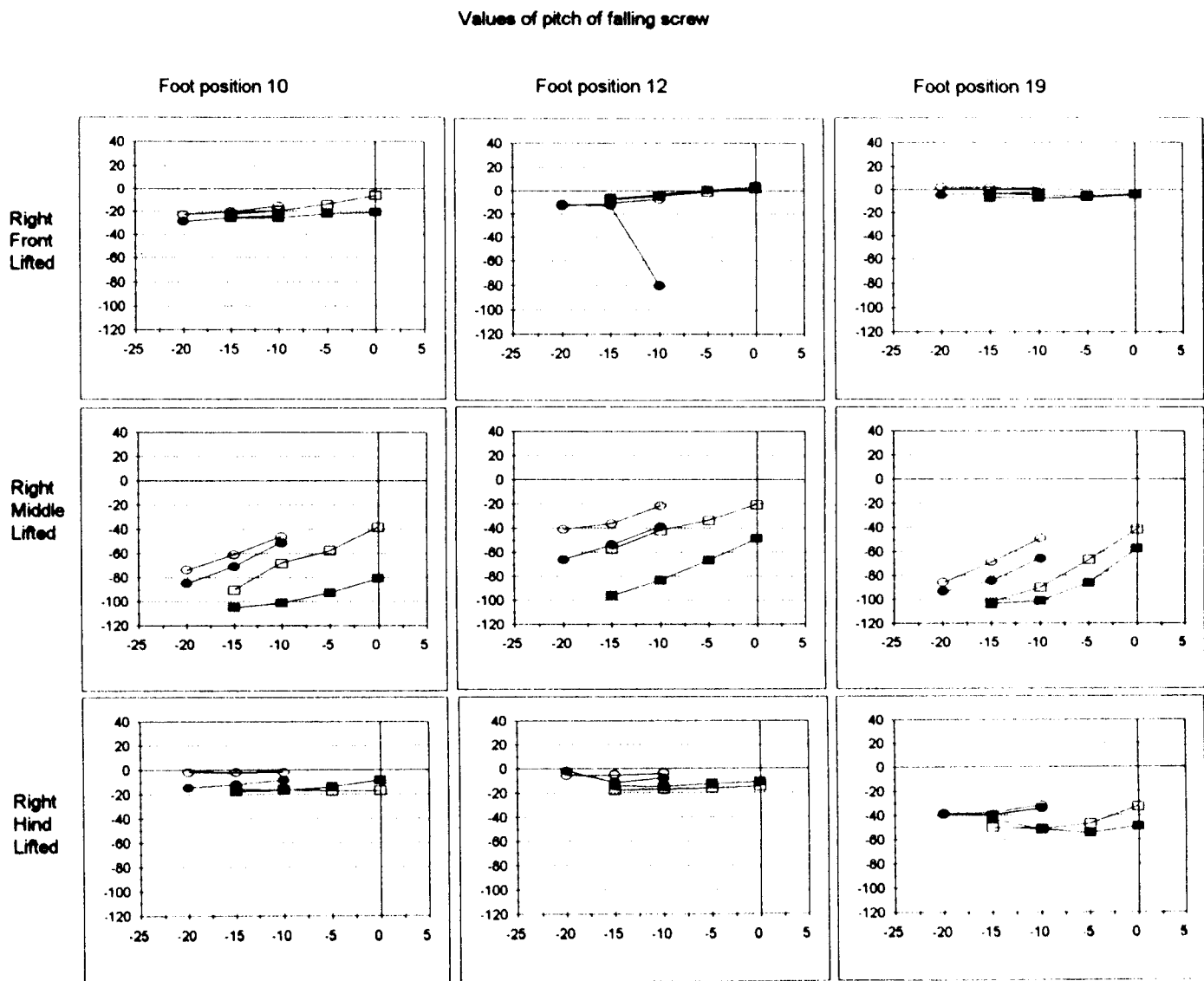


Figure 5.9 : Variation of pitch of screw axis.

From Figure 5.3 through 5.9 following conclusions can be drawn :

- (i) In Figure 5.3 it can be seen that value of E_x or X component of unit vector of screw axis was negative for all the cases, which is because of the reason that the right legs of machine are lifted. The X component of screw axis was closer to -1 for foot position 10 and 12, which is similar to what is wanted, as described in case II. As explained earlier the screw parameters were found for a simultaneous increase in magnitude of pitch of machine and increase in height of mass center of machine, that is for machine parameters having similar body workspace, the X component of unit vector of screw axis approaches -1 with decrease in pitch and decrease in height of mass center of machine.
- (ii) It can be seen in Figure 5.4 that Y component was positive when right front and right middle leg is lifted and is negative when right hind leg is lifted. The value of E_y does not change a lot.
- (iii) It was seen that the Z component of unit vector of screw axis was positive for all the cases (Figure 5.5). Also the Z component was closer to zero for foot position 12 as compared to foot position 10 and 19, for lifting any of the leg. The value of E_z approaches zero with simultaneous decrease in magnitude of pitch of machine and decrease in height of mass center of machine.

(iv) The pitch of the falling screw of machine was negative for all the cases (Figure 5.9). The value of pitch of falling screw is closer to zero when right front or right hind leg is lifted. It can be seen that there is sudden change in value of pitch of screw when right front leg is lifted at foot position 12 and pitch of machine at -10 degrees. The sudden change is because of very small magnitude of both the rotation about screw axis and translation along screw axis, which means that the sudden change is only an artifact resulting from round off error. The case is also shown in

Figure 5.10.

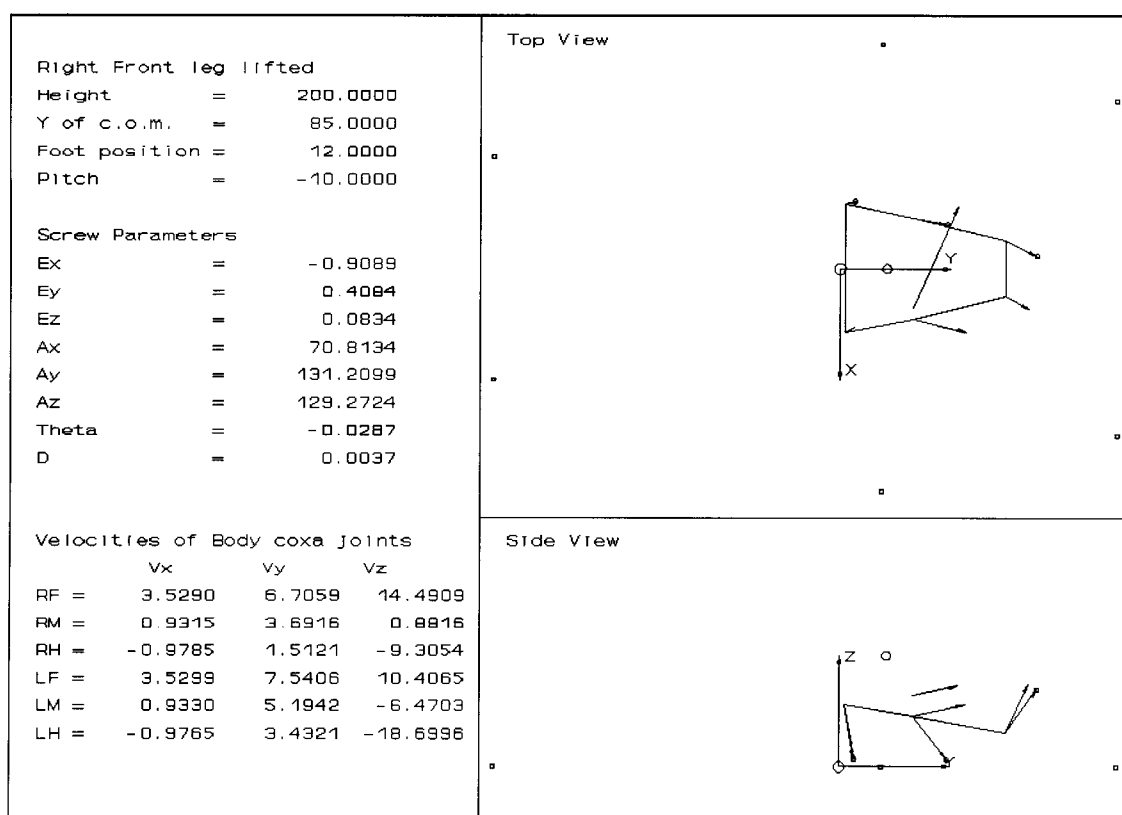


Figure 5.10 : Plot showing falling screw of machine

(v) There is very small change in E_y or Y component of unit vector of screw axis as compared to change in X and Z component of unit vector of screw axis. The increase in E_x is compensated by decrease in E_z and there is very small change in E_y . As a result the screw axis becomes more parallel to xy_g plane. Since the magnitude of E_x is larger than E_y the screw axis is more aligned along negative x_g axis.

(vi) In most cases the value of A_x or X component of perpendicular to screw axis approaches zero for the cases. These values of A_x are in ground coordinate system.

(vii) The Value of A_z or Z component of perpendicular to screw axis is not relatively large as compared to height of center of mass of machine, it decreases with decrease in height of center of mass.

It was seen in the above discussion that the E_y or the Y component of unit vector of screw axis, is not close to one. Also in case I, it was discussed that the falling motion of machine can be used if E_y is large, that is, the screw axis is parallel to y_g axis. Thus case I is not possible for the machine.

It was also discussed in case II, that the falling motion of machine can be used if E_x or X component of unit vector of screw axis is large. It was

seen that the E_x is close to -1. The other requirement for satisfying case II is that the pitch of the screw is small, which is also satisfied since the pitch of screw in most of the cases is close to zero and is negative. As a result the combination of negative E_x and negative pitch of screw shows that the machine has a tendency to fall in the forward direction, or make a curve along the positive y_g axis. The third requirement of the case was that A_z or the Z component of perpendicular to screw axis is large or at infinity, which is not satisfied since the A_z is relatively small as compared to height of body.

Since neither case I nor case II is satisfied, the next step would be to use falling motion of machine in the best possible way, to make the body move in the required direction with specified velocity. As discussed by Fichter et al 1991, the direction and intensity of falling screw can be controlled by changing leg lengths. As shown in figure 5.2, the falling screw of machine results in velocities of points on the machine. Since the body-coxa joints are on the machine, their velocities are known. If desired body twist is known, then desired velocity of each Hooke joint center is also known then difference between actual and desired velocity can be calculated. The velocity difference at the hooke joints can be provided by extending or contracting the leg connection to the hooke joint. Since the foot is on the ground the velocity difference should be in the direction from the foot to hooke joint.

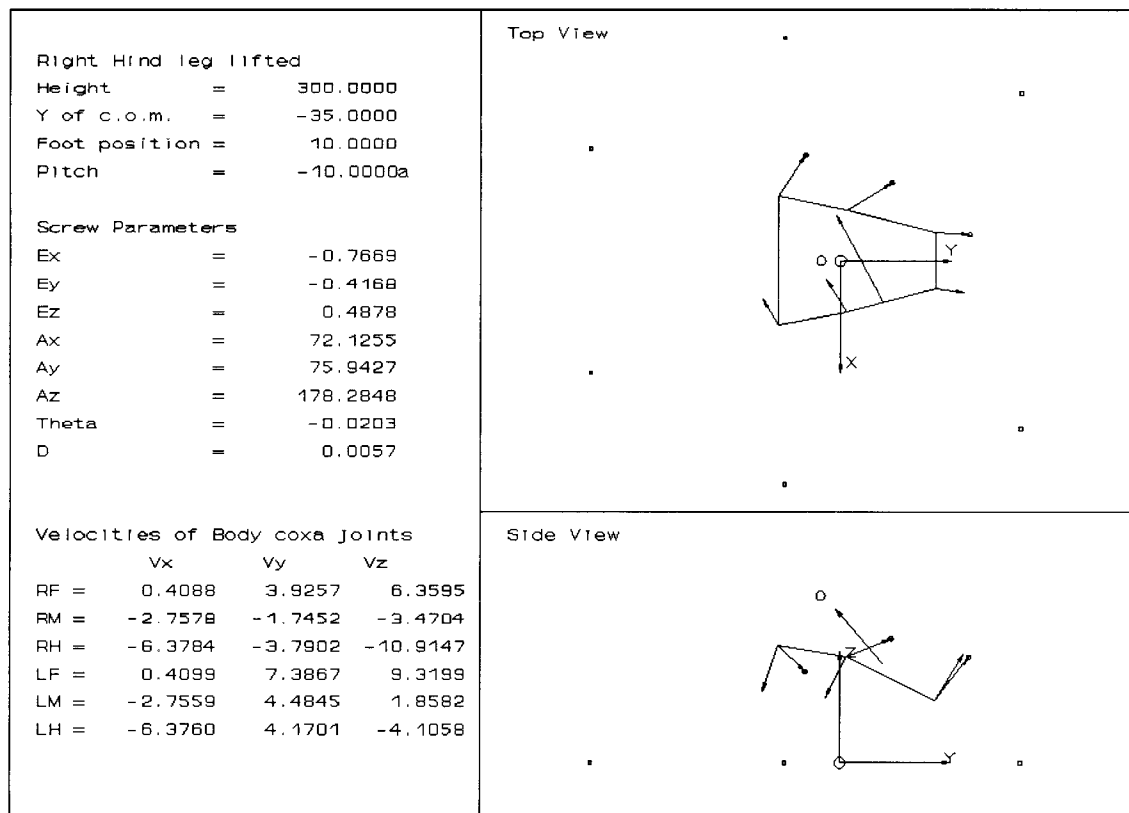


Figure 5.11 : Plot showing falling screw of machine

Figure 5.11 shows screw parameters for a particular set of machine parameters. For this case, the velocity difference required for the right front leg at the body-coxa joints can be calculated as :

$$b1 = -0.5835 e_1 + -0.7693 e_2 + 0.2602 e_3$$

$$b2 = -0.4088 e_1 + 3.9257 e_2 + 6.3595 e_3$$

$$a.(b1) + b2 = d e_1 + c e_2 + 0 e_3$$

where b1 is a unit vector from foot to ground for front leg and b2 is

the velocity at the front body coxa joints, and a, c, d are constants. In the above equation it is assumed that desired velocity is in plane parallel to xy_g plane.

Solving for a, c, d we get :

$$a = 24.441$$

$$c = -14.87$$

$$d = -13.852$$

Thus for this example the velocity produced by the extension in the front leg will be in negative x_g and negative y_g direction. Similar calculations can be done for other legs.

6. Suggestion for Future Studies

Investigation of body workspace and falling screw of a six legged walking machine for fixed foot position is still under development, and a number of aspects of this area of study require further attention. This chapter includes recommendations for future study of selected areas related to the subject of current investigation.

Through out the study all the joints were assumed to be ideal, that is friction at the joints was altogether neglected. Also it was assumed that there is no slippage between foot and ground. The body workspace of machine may change with inclusion of slippage at the ground and may result in change of foot position of the machine. It would also be interesting to investigate the change in screw parameters with inclusion of friction at the joints.

It was seen in chapter 4 that in some cases the kinematic workspace of machine was limiting the body workspace. Since the maximum and minimum values of joints angles, which is deciding factor for kinematic workspace, were taken similar to that of beetle, the body workspace of machine may increase with increase in the range of joint angles of machine.

The variation of body workspace of machine was studied at symmetrical foot positions only. Since the machine is going to lift only one leg at a time, asymmetry in foot position will be reached, and it is suggested that study of variation of body workspace of machine with asymmetric foot positions be also done.

For this study, effect of removal of one constraint on walking machine was considered. Since it is possible to put one additional constraint on any of Hooke joint (body-coxa joint), investigation of choosing the joint that is most nearly reciprocal to the falling twist and can actively control the joint, can be done.

This study examined the effect of foot position, pitch of machine and height of machine on body workspace and screw parameters. Roll and yaw of machine were completely neglected. The variation of screw parameters and body workspace of machine with changes in roll and yaw of machine will be helpful in studying the turning of machine.

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Appendix

A1 Information Files

As explained in chapter 3 the system description file for pendulum was written. After SD/FAST is run there are number of files which are created. `pend_i`, that is, information file is the file which contains the information about the model for which it generated equations. The information file for the pendulum is shown below. The file is separated in three parts : Roadmap, State Index Map and System parameters.

The Roadmap is the tabular representation of system topology. First the tree system is described, followed by description of the loop joints. Each joint and body is numbered, the type of joint is shown, and the inboard and outboard bodies involved are indicated. The configuration and velocity of a system are represented by a state vector, composed of position coordinates (q 's) followed by velocity variables (u 's). The state index map shows the correspondence between the entries in the state and the associated joint number or axis number pair, which is useful when user writes code for doing any kind of simulation. The system parameters simply list the system parameters, such as number of bodies and joints, number of constraints, etc. These parameters are important to know since arrays are typically dimensioned to these values.

A1.1 Information File For Pendulum

SD/FAST Information File: pend.sd

Generated 02-Aug-1992 22:15:05 by SD/FAST, Kane's formulation
(sdfast BX.2.1 #30447) on machine ID 14140a43

ROADMAP (pend.sd)

Bodies	Inb			
No	Name	body	Joint type	Coords q
-1	\$ground			
0	pendulum	-1	Pin	0

STATE INDEX TO JOINT/AXIS MAP (pend.sd)

Index						
q u	Joint	Axis	Joint type	Axis type	Joint Name	
0 1	0	0	Pin	rotate		

SYSTEM PARAMETERS (pend.sd)

Parameter	Value	Description
-----------	-------	-------------

nbod	1	no. bodies (also, no. of tree joints)
njnt	1	total number of joints (tree + loop)
ndof	1	no. degrees of freedom allowed by tree joints
nloop	0	no. loop joints
nldof	0	no. degrees of freedom allowed by loop joints
nq	1	no. position coordinates in state (tree joints)
nu	1	no. rate coordinates in state (tree joints)
nlq	0	no. position coordinates describing loop joints
nlu	0	no. rate coordinates describing loop joints
nc	0	total no. constraints defined
nlc	0	no. loop joint constraints
npresc	0	no. prescribed motion constraints
nuserc	0	no. user constraints

nq	3	no. position coordinates in state (tree joints)
nu	3	no. rate coordinates in state (tree joints)
nlq	1	no. position coordinates describing loop joints
nlu	1	no. rate coordinates describing loop joints
nc	5	total no. constraints defined
nlc	5	no. loop joint constraints
npresc	0	no. prescribed motion constraints
nuserc	0	no. user constraints

A1.3 Information File For Machine

SD/FAST Information File: mac1.sd

Generated 22-May-1993 18:08:30 by SD/FAST, Kane's formulation
(sdfast BX.2.1 #30447) on machine ID 14140a43

ROADMAP (mac1.sd)

Bodies		Inb										
No	Name	body	Joint	type	Coords	q	Multipliers					

-1	\$ground											
0	table	-1	Sixdof		0?	1?	2?					
		...			3?	4?	5?	24	0p	1p	2p	3p 4p 5p
1	rifrcoxa	0	Pin		6?				6p			
2	rifrfemure	1	Pin		7?				7p			
3	rifrtibia	2	Pin		8?				8p			
4	rimicoxa	0	Pin		9?				9p			
5	rimifemure	4	Pin		10?				10p			
6	rimitibia	5	Pin		11?				11p			
7	rihicoxa	0	Pin		12?				12p			
8	rihifemure	7	Pin		13?				13p			
9	rihitibia	8	Pin		14?				14p			
10	lefrcoxa	0	Pin		15?				15p			
11	lefrfemure	10	Pin		16?				16p			
12	lefrtibia	11	Pin		17?				17p			
13	lemicoxa	0	Pin		18?				18p			
14	lemifemure	13	Pin		19?				19p			
15	lemitibia	14	Pin		20?				20p			
16	lehicoxa	0	Pin		21?				21p			
17	lehifemure	16	Pin		22?				22p			
18	lehitibia	17	Pin		23?				23p			

Loop Joints			Pseudo Coords lq									
19	rifrtibia	-1	Ball	0?	1?	2?	18	24p	25p	26p	42	43 44
20	rimitibia	-1	Ball	3?	4?	5?	19	27p	28p	29p	45	46 47
21	rihitibia	-1	Ball	6?	7?	8?	20	30p	31p	32p	48	49 50
22	lefrtibia	-1	Ball	9?	10?	11?	21	33p	34p	35p	51	52 53
23	lemitibia	-1	Ball	12?	13?	14?	22	36p	37p	38p	54	55 56
24	lehitibia	-1	Ball	15?	16?	17?	23	39p	40p	41p	57	58 59

STATE INDEX TO JOINT/AXIS MAP (mac1.sd)

Index		Joint	Axis	Joint type	Axis type	Joint Name
q u						
0 25	0	0?	Sixdof		translate	
1 26	.	1?	.		translate	
2 27	.	2?	.		translate	
3 28	.	3?	.		quaternion	
4 29	.	4?	.		quaternion	
5 30	.	5?	.		quaternion	
6 31	1	0?	Pin		rotate	
7 32	2	0?	Pin		rotate	
8 33	3	0?	Pin		rotate	
9 34	4	0?	Pin		rotate	
10 35	5	0?	Pin		rotate	
11 36	6	0?	Pin		rotate	
12 37	7	0?	Pin		rotate	
13 38	8	0?	Pin		rotate	
14 39	9	0?	Pin		rotate	
15 40	10	0?	Pin		rotate	
16 41	11	0?	Pin		rotate	
17 42	12	0?	Pin		rotate	
18 43	13	0?	Pin		rotate	
19 44	14	0?	Pin		rotate	
20 45	15	0?	Pin		rotate	
21 46	16	0?	Pin		rotate	
22 47	17	0?	Pin		rotate	
23 48	18	0?	Pin		rotate	
24	0	6	Sixdof		4th quat	

lq|lu

0 24	0	0?	Ball	quaternion
1 25	.	1?	.	quaternion
2 26	.	2?	.	quaternion
3 27	1	0?	Ball	quaternion
4 28	.	1?	.	quaternion
5 29	.	2?	.	quaternion
6 30	2	0?	Ball	quaternion
7 31	.	1?	.	quaternion
8 32	.	2?	.	quaternion
9 33	3	0?	Ball	quaternion
10 34	.	1?	.	quaternion
11 35	.	2?	.	quaternion

12 36	4	0?	Ball	quaternion
13 37	.	1?	.	quaternion
14 38	.	2?	.	quaternion
15 39	5	0?	Ball	quaternion
16 40	.	1?	.	quaternion
17 41	.	2?	.	quaternion
18	0	3	Ball	4th quat
19	1	3	Ball	4th quat
20	2	3	Ball	4th quat
21	3	3	Ball	4th quat
22	4	3	Ball	4th quat
23	5	3	Ball	4th quat

SYSTEM PARAMETERS (mac1.sd)

Parameter Value Description

nbod	19	no. bodies (also, no. of tree joints)
njnt	25	total number of joints (tree + loop)
ndof	24	no. degrees of freedom allowed by tree joints
nloop	6	no. loop joints
nldof	18	no. degrees of freedom allowed by loop joints
nq	25	no. position coordinates in state (tree joints)
nu	24	no. rate coordinates in state (tree joints)
nlq	24	no. position coordinates describing loop joints
nlu	18	no. rate coordinates describing loop joints
nc	60	total no. constraints defined
nlc	18	no. loop joint constraints
npresc	42	no. prescribed motion constraints
nuserc	0	no. user constraints

A2 System Description File For Machine

The system description file for machine is shown below. In system description file the following nomenclature is used :

table	: main body of machine
rifrcoxa	: right front coxa
rifrfemur	: right front femur
rifrtibia	: right front tibia
lemicoxa	: left middle coxa
lemifemur	: left middle femur
lemitibia	: left middle tibia
etc...	

The first two alphabets indicate whether it is right leg or left leg being described; the next two alphabets indicate the front, middle or hind leg, and in the last is the name of the segment of the leg being described.

The question marks in the file mean that the number can be changed within the user written code. As explained in chapter 3 the system description file for the machine was developed by using the data from the A-Model parameters of the machine.

```
# SYSTEM DESCRIPTION FILE FOR MACHINE
# FILE NAME = mac1.sd
#
```

```
gravity = 0.0? 0.0? -9800.0?
language = c
```

```
#          DEFINING THE BODY OF THE MACHINE
# THE '?' INDICATES THAT THE NUMBERS CAN BE CHANGED FROM THE
#          'C' CODE WRITTEN
```

```
body = table
inb = $ground
joint = sixdof
mass = 18900.0?
inertia = 1.26e9? 0.0? 0.0?
          0.0? 2.9141e8? 0.0?
          0.0? 0.0? 1.535e9?
bodytojoint = 0.0? 0.0? 0.0?
inbtojoint = 0.0? 0.0? 400.0?
pin = 1.0? 0.0? 0.0?
pin = 0.0? 1.0? 0.0?
pin = 0.0? 0.0? 1.0?
prescribed = 0? 0? 0? 0? 0? 0?
```

```
# DEFINING THE RIGHT FRONT LEG OF THE MACHINE
```

```
# The coxa of the right front leg of the machine which is connected to the
# body by the revolute joint.
# rfrcoxa = right front coxa
```

```
body = rfrcoxa
inb = table
joint = pin
mass = 30.0?
inertia = 34700.0? 0.0? 0.0?
          0.0? 34700.0? 0.0?
          0.0? 0.0? 900.0?
bodytojoint = 0.0? 0.0? 0.0?
inbtojoint = 50.0? 235.0? -100.0?
pin = -0.5? 0.0? -0.866?
prescribed = 0?
```

```
# The femur of the right front leg of the machine which is connected to the
# coxa by the revolute joint.
# rirrfemure = right front femur
```

```
body = rirrfemure
inb = rifrcoxa
joint = pin
mass = 170.0?
inertia = 2.275e6? 0.0? 0.0?
          0.0? 4.89e4? 0.0?
          0.0? 0.0? 2.275e6?
bodytojoint = 0.0? -139.0? 0.0?
inbtojoint = 0.0? 0.0? 0.0?
pin = 0.866? 0.0? -0.5?
prescribed = 0?
```

```
# The tibia of the right front leg of the machine which is connected to the
# femur by the revolute joint.
# rifrtibia = right front tibia
```

```
body = rifrtibia
inb = rirrfemure
joint = pin
mass = 111.0?
inertia = 9.92e5? 0.0? 0.0?
          0.0? 4.07e4? 0.0?
          0.0? 0.0? 9.92e5?
bodytojoint = 0.0? -87.0? 0.0?
inbtojoint = 0.0? 161.0? 0.0?
pin = 0.866? 0.0? -0.5?
prescribed = 0?
```

```
# The tibia of the right front leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# rifrtibia = right front tibia
```

```
body = rifrtibia
inb = $ground
joint = ball
bodytojoint = 0.0? 163.0? 0.0?
inbtojoint = 300.0? 325.0? 0.0?
prescribed = 0? 0? 0?
```

DEFINING THE RIGHT MIDDLE LEG OF THE MACHINE

The coxa of the right middle leg of the machine which is connected to the
body by the revolute joint.

rimicoxa = right middle coxa

body = rimicoxa

inb = table

joint = pin

mass = 30.0?

inertia = 3.47e4? 0.0? 0.0?

0.0? 3.47e4? 0.0?

0.0? 0.0? 900.0?

bodytojoint = 0.0? 0.0? 0.0?

inbtojoint = 90.0? 65.0? -100.0?

pin = -0.433? 0.25? -0.866?

prescribed = 0?

The femur of the right middle leg of the machine which is connected to
the

coxa by the revolute joint.

rimifemure = right middle femur

body = rimifemure

inb = rimicoxa

joint = pin

mass = 190.0?

inertia = 2.44e6? 1.38e6? 0.0?

1.38e6? 8.47e5? 0.0?

0.0? 0.0? 3.23e6?

bodytojoint = -80.0? -138.56? 0.0?

inbtojoint = 0.0? 0.0? 0.0?

pin = -0.75? 0.433? 0.5?

prescribed = 0?

```
# The tibia of the right middle leg of the machine which is connected to the
# femur by the revolute joint.
# rimitibia = right middle tibia
```

```
body = rimitibia
inb = rimifemure
joint = pin
mass = 141.0?
inertia = 1.577e6? 8.86e5? 0.0?
          8.86e5? 5.54e5? 0.0?
          0.0? 0.0? 2.089e6?
bodytojoint = -67.5? -116.91? 0.0?
inbtojoint = 95.0? 164.544? 0.0?
pin = -0.75? 0.433? 0.5?
prescribed = 0?
```

```
# The tibia of the right middle leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# rimitibia = right middle tibia
```

```
body = rimitibia
inb = $ground
joint = ball
bodytojoint = 107.5? 186.195? 0.0?
inbtojoint = 400.0? -100.0? 0.0?
prescribed = 0? 0? 0?
```

```
# DEFINING THE RIGHT HIND LEG OF THE MACHINE
```

```
# The coxa of the right hind leg of the machine which is connected to the
# body by the revolute joint.
# rihicoxa = right hind coxa
```

```
body = rihicoxa
inb = table
joint = pin
mass = 30.0?
inertia = 3.47e4? 0.0? 0.0?
          0.0? 3.47e4? 0.0?
          0.0? 0.0? 900?
bodytojoint = 0.0? 0.0? 0.0?
inbtojoint = 115.0? -60.0? -100.0?
pin = -0.5? 0.5? -0.707?
prescribed = 0?
```



```
# The femur of the right hind leg of the machine which is connected to the
# coxa by the revolute joint.
# rihifemure = right hind femur
```

```
body = rihifemure
inb = rihicoxa
joint = pin
mass = 225.0?
inertia = 2.958e6? 2.906e6? 0.0?
          2.906e6? 2.958e6? 0.0?
          0.0? 0.0? 5.865e6?
bodytojoint = -144.956? -144.956? 0.0?
inbtojoint = 0.0? 0.0? 0.0?
pin = -0.5? 0.5? 0.707?
prescribed = 0?
```

```
# The tibia of the right hind leg of the machine which is connected to the
# femur by the revolute joint.
# rihitibia = right hind tibia
```

```
body = rihitibia
inb = rihifemure
joint = pin
mass = 156.0?
inertia = 1.462e6? 1.419e6? 0.0?
          1.419e6? 1.462e6? 0.0?
          0.0? 0.0? 2.881e6?
bodytojoint = -113.137? -113.137? 0.0?
inbtojoint = 173.24? 173.24? 0.0?
pin = -0.5? 0.5? 0.707?
prescribed = 0?
```

```
# The tibia of the right hind leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# rihitibia = right hind tibia
```

```
body = rihitibia
inb = $ground
joint = ball
bodytojoint = 169.7? 169.7? 0.0?
inbtojoint = 200.0? -450.0? 0.0?
prescribed = 0? 0? 0?
```

DEFINING THE LEFT FRONT LEG OF THE MACHINE

The coxa of the left front leg of the machine which is connected to the
body by the revolute joint.

lefrcoxa = left front coxa

body = lefrcoxa

inb = table

joint = pin

mass = 30.0?

inertia = 3.47e4? 0.0? 0.0?

0.0? 3.47e4? 0.0?

0.0? 0.0? 900?

bodytojoint = 0.0? 0.0? 0.0?

inbtojoint = -50.0? 235.0? -100.0?

pin = 0.5? 0.0? -0.866?

prescribed = 0?

The femur of the left front leg of the machine which is connected to the
coxa by the revolute joint.

lefrfemure = left front femur

body = lefrfemure

inb = lefrcoxa

joint = pin

mass = 170.0?

inertia = 2.275e6? 0.0? 0.0?

0.0? 4.89e4? 0.0?

0.0? 0.0? 2.275e6?

bodytojoint = 0.0? -139.0? 0.0?

inbtojoint = 0.0? 0.0? 0.0?

pin = -0.866? 0.0? -0.5?

prescribed = 0?

```
# The tibia of the left front leg of the machine which is connected to the
# femur by the revolute joint.
# leftrtibia = left front tibia
```

```
body = leftrtibia
inb = lefrfemure
joint = pin
mass = 111.0?
inertia = 9.92e5? 0.0? 0.0?
          0.0? 4.07e4? 0.0?
          0.0? 0.0? 9.92e5?
bodytojoint = 0.0? -87.0? 0.0?
inbtojoint = 0.0? 161.0? 0.0?
pin = -0.866? 0.0? -0.5?
prescribed = 0?
```

```
# The tibia of the left front leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# leftrtibia = left front tibia
```

```
body = leftrtibia
inb = $ground
joint = ball
bodytojoint = 0.0? 163.0? 0.0?
inbtojoint = -300.0? 325.0? 0.0?
prescribed = 0? 0? 0?
```

DEFINING THE LEFT MIDDLE LEG OF THE MACHINE

```
# The coxa of the left middle leg of the machine which is connected to the
# body by the revolute joint.
# lemicoxa = left middle coxa
```

```
body = lemicoxa
inb = table
joint = pin
mass = 30.0?
inertia = 3.47e4? 0.0? 0.0?
          0.0? 3.47e4? 0.0?
          0.0? 0.0? 900?
bodytojoint = 0.0? 0.0? 0.0?
inbtojoint = -90.0? 65.0? -100.0?
pin = 0.433? 0.25? -0.866?
prescribed = 0?
```

```
# The femur of the left middle leg of the machine which is connected to the
# coxa by the revolute joint.
# lemifemure = left middle femur
```

```
body = lemifemure
inb = lemicoxa
joint = pin
mass = 190.0?
inertia = 2.44e6? 1.38e6? 0.0?
          1.38e6? 8.47e5? 0.0?
          0.0? 0.0? 3.23e6?
bodytojoint = 80.0? -138.56? 0.0?
inbtojoint = 0.0? 0.0? 0.0?
pin = 0.75? 0.433? 0.5?
prescribed = 0?
```

```
# The tibia of the left middle leg of the machine which is connected to the
# femur by the revolute joint.
# lemitibia = left middle tibia
```

```
body = lemitibia
inb = lemifemure
joint = pin
mass = 141.0?
inertia = 1.577e6? 8.86e5? 0.0?
          8.86e5? 5.54e5? 0.0?
          0.0? 0.0? 2.089e6?
bodytojoint = 67.5? -116.91? 0.0?
inbtojoint = -95.0? 164.544? 0.0?
pin = 0.75? 0.433? 0.5?
prescribed = 0?
```

```
# The tibia of the left middle leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# lemitibia = left middle tibia
```

```
body = lemitibia
inb = $ground
joint = ball
bodytojoint = -107.5? 186.195? 0.0?
inbtojoint = -400.0? -100.0? 0.0?
prescribed = 0? 0? 0?
```

DEFINING THE LEFT HIND LEG OF THE MACHINE

The coxa of the left hind leg of the machine which is connected to the
 # body by the revolute joint.
 # lexicoxa = left hind coxa

```
body = lexicoxa
inb = table
joint = pin
mass = 30.0?
inertia = 3.47e4? 0.0? 0.0?
          0.0? 3.47e4? 0.0?
          0.0? 0.0? 900?
bodytojoint = 0.0? 0.0? 0.0?
inbtojoint = -115.0? -60.0? -100.0?
pin = 0.5? 0.5? -0.707?
prescribed = 0?
```

The femur of the left hind leg of the machine which is connected to the
 # coxa by the revolute joint.
 # lehifemure = left hind femur

```
body = lehifemure
inb = lexicoxa
joint = pin
mass = 225.0?
inertia = 2.958e6? 2.906e6? 0.0?
          2.906e6? 2.958e6? 0.0?
          0.0? 0.0? 5.865e6?
bodytojoint = 144.956? -144.956? 0.0?
inbtojoint = 0.0? 0.0? 0.0?
pin = 0.5? 0.5? 0.707?
prescribed = 0?
```

```
# The tibia of the left hind leg of the machine which is connected to the
# femur by the revolute joint.
# lehitibia = left hind tibia
```

```
body = lehitibia
inb = lehifemure
joint = pin
mass = 156.0?
inertia = 1.462e6? 1.419e6? 0.0?
          1.419e6? 1.462e6? 0.0?
          0.0? 0.0? 2.889e6?
bodytojoint = 113.137? -113.137? 0.0?
inbtojoint = -173.24? 173.24? 0.0?
pin = 0.5? 0.5? 0.707?
prescribed = 0?
```

```
# The tibia of the left hind leg of the machine which is connected to the
# ground by the ball joint & also it is the loop joint.
# lehitibia = left hind tibia
```

```
body = lehitibia
inb = $ground
joint = ball
bodytojoint = -169.7? 169.7? 0.0?
inbtojoint = -200.0? -450.0? 0.0?
prescribed = 0? 0? 0?
```

A3 Moment of Inertia of Machine and Segments of Legs

For SD/FAST the inertia matrix of each body is specified about its mass center, in the body coordinate frame. For any coordinate frame with unit vector subscripts x, y, z , the inertia matrix I is given by :

$$I = \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ I_{yx} & I_y & I_{yz} \\ I_{zx} & I_{zy} & I_z \end{bmatrix} \quad \text{..(a1)}$$

where :

$$I_x = \int (y^2 + z^2)dm \quad \text{..(a2)}$$

$$I_{xy} = - \int (xy)dm = I_{yx} \quad \text{..(a3)}$$

$$I_{xz} = - \int (xz)dm = I_{zx} \quad \text{..(a4)}$$

$$I_y = \int (x^2 + z^2)dm \quad \text{..(a5)}$$

$$I_{yz} = - \int (yz)dm = I_{zy} \quad \text{..(a6)}$$

$$I_z = \int (x^2 + y^2)dm \quad \text{..(a7)}$$

where x, y, z is the distance from the mass center of element dm along the axis in the direction of x, y, z axis of the body coordinate system.

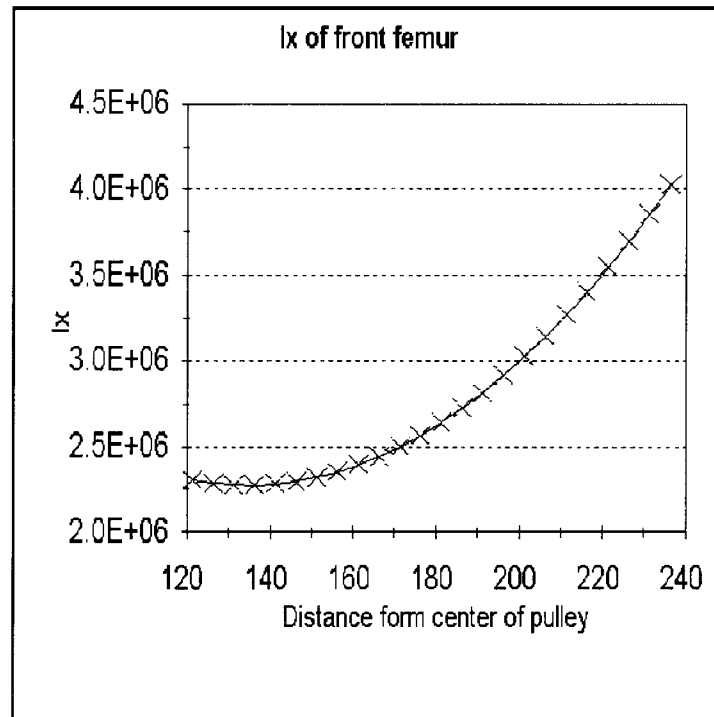


Figure A1 : Variation of I_x with distance from axis for front femur.

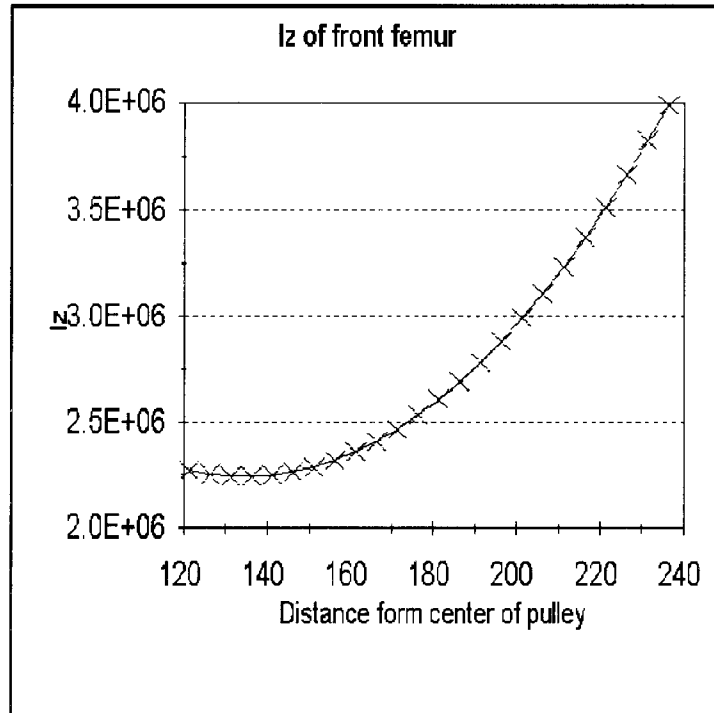


Figure A2 : Variation of I_z with distance from axis for front femur.

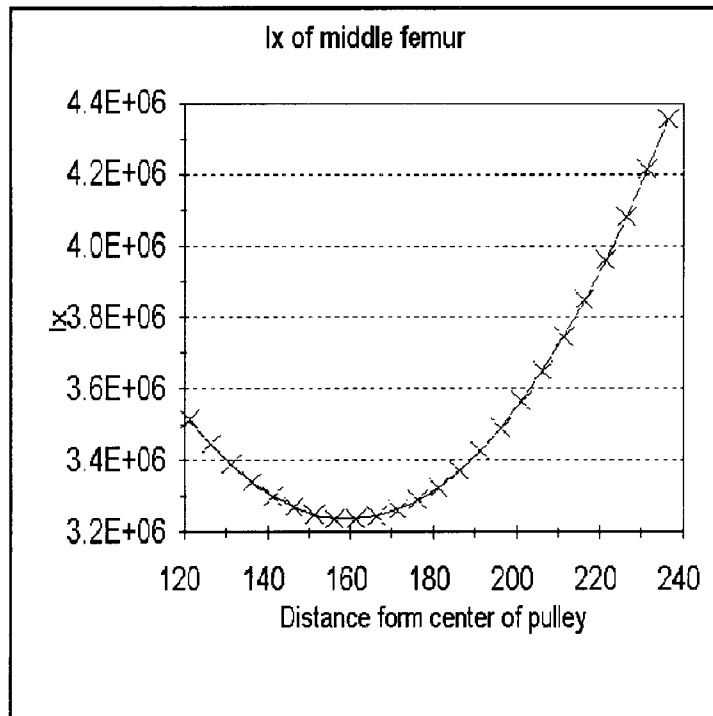


Figure A3 : Variation of I_x with distance from axis for middle femur.

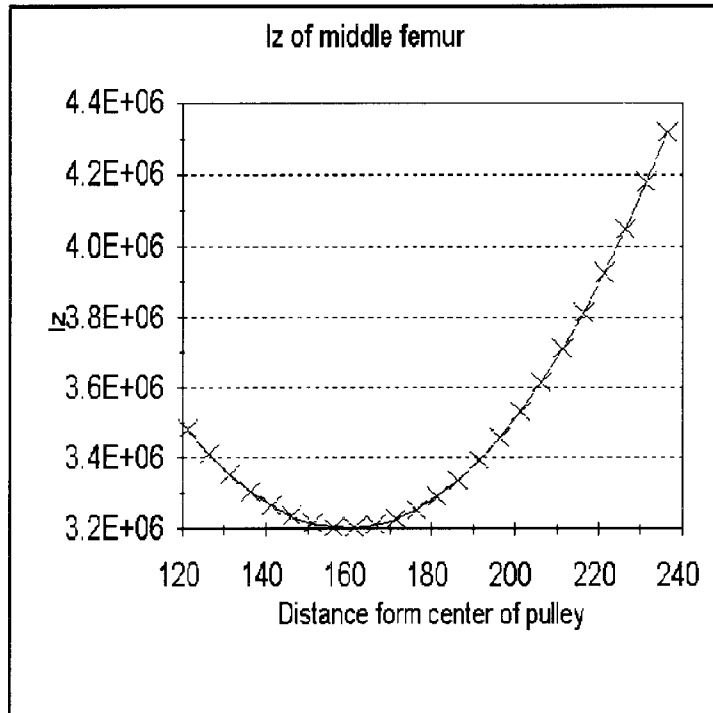


Figure A4 : Variation of I_z with distance from axis for middle femur.

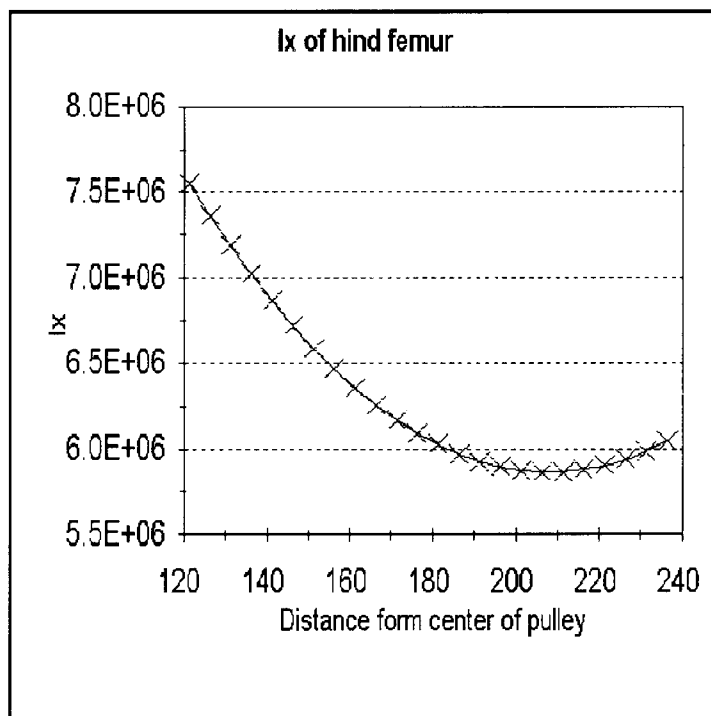


Figure A5 : Variation of I_x with distance from axis for hind femur.

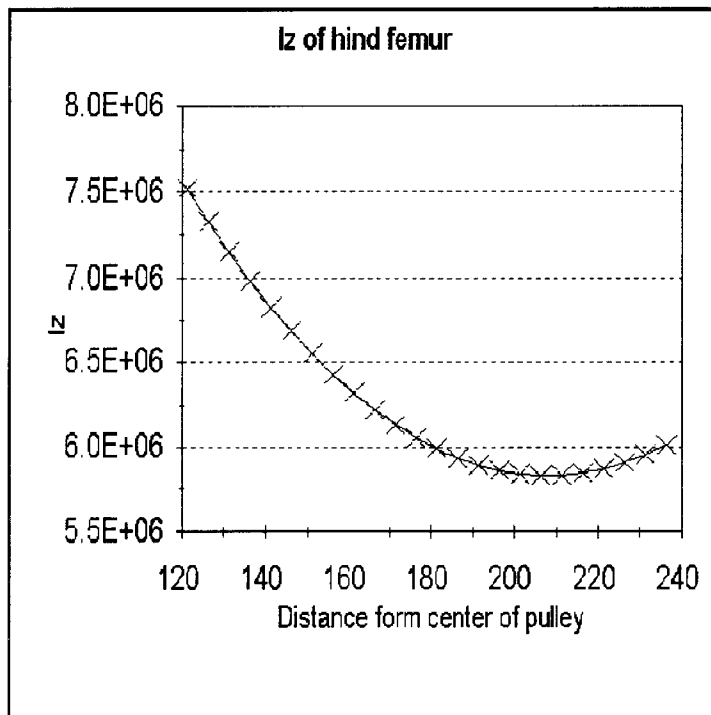


Figure A6 : Variation of I_z with distance from axis for hind femur.

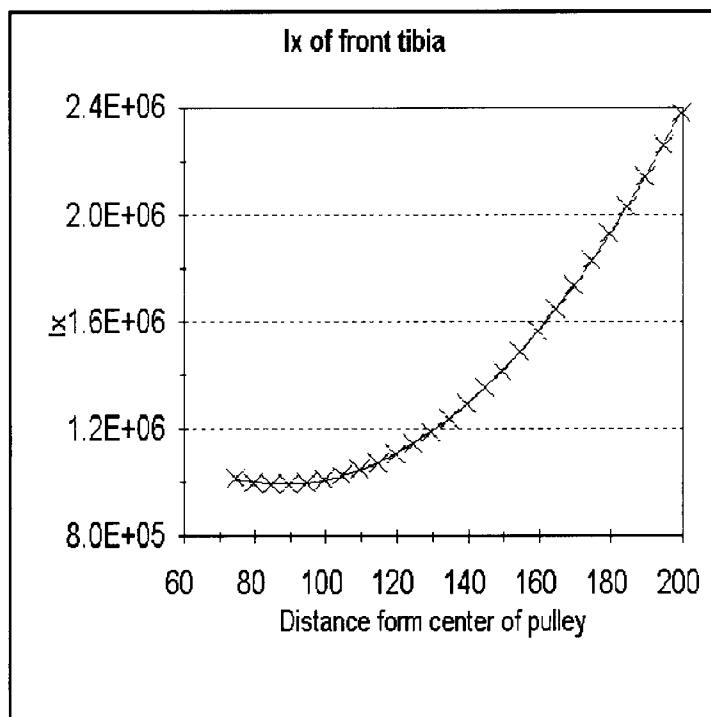


Figure A7 : Variation of I_x with distance from axis for front tibia.

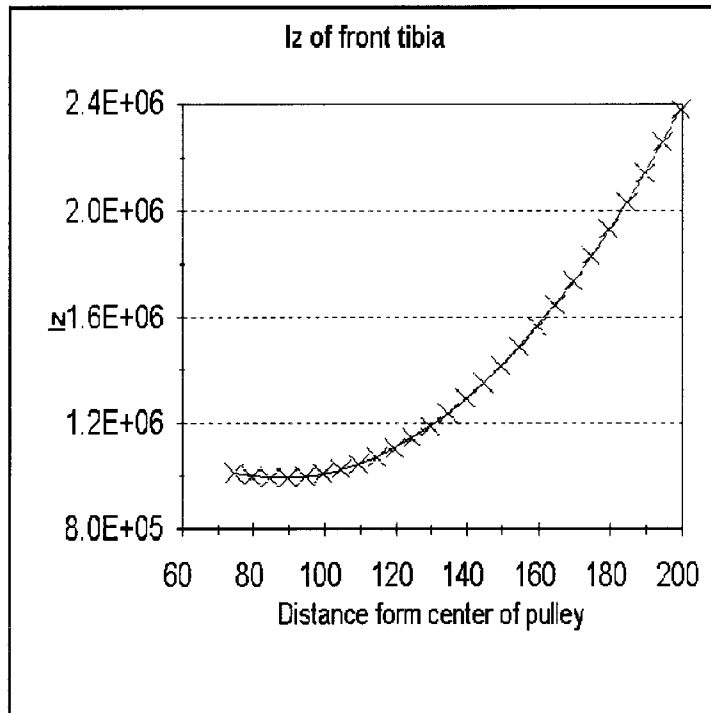


Figure A8 : Variation of I_z with distance from axis for front tibia.

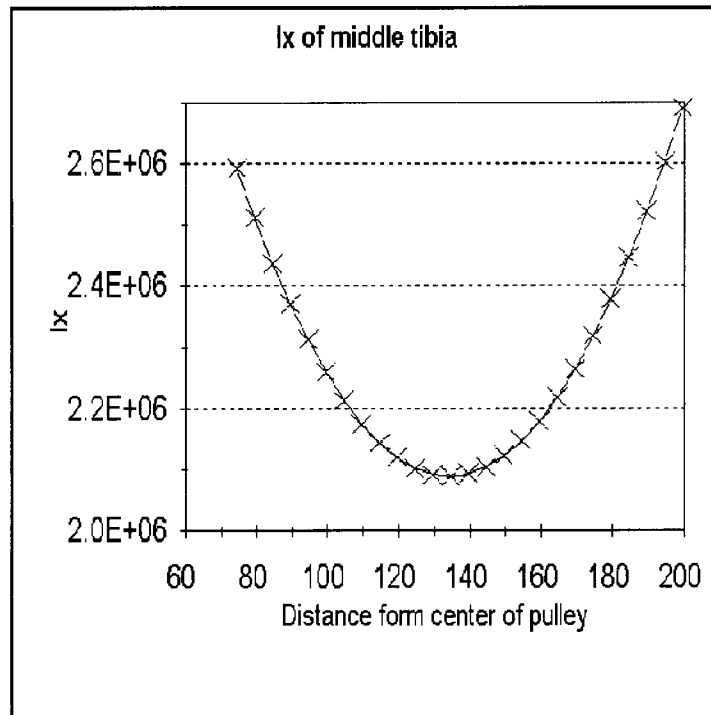


Figure A9 : Variation of I_x with distance from axis for middle tibia.

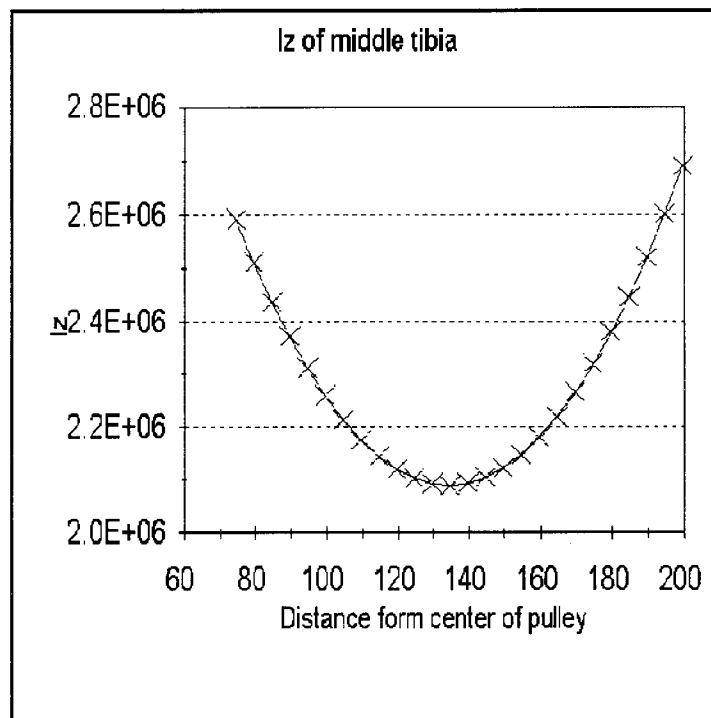


Figure A10 : Variation of I_z with distance from axis for middle tibia.

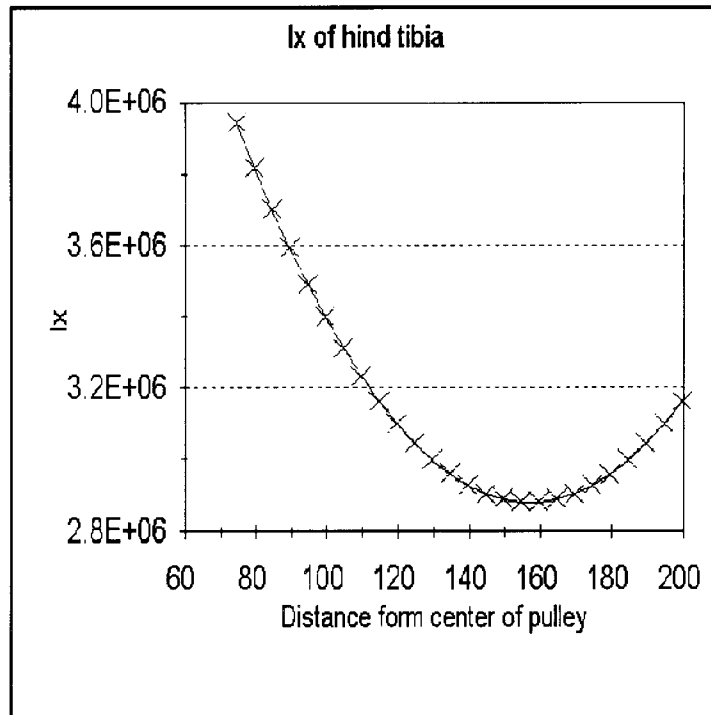


Figure A11 : Variation of I_x with distance from axis for hind tibia.

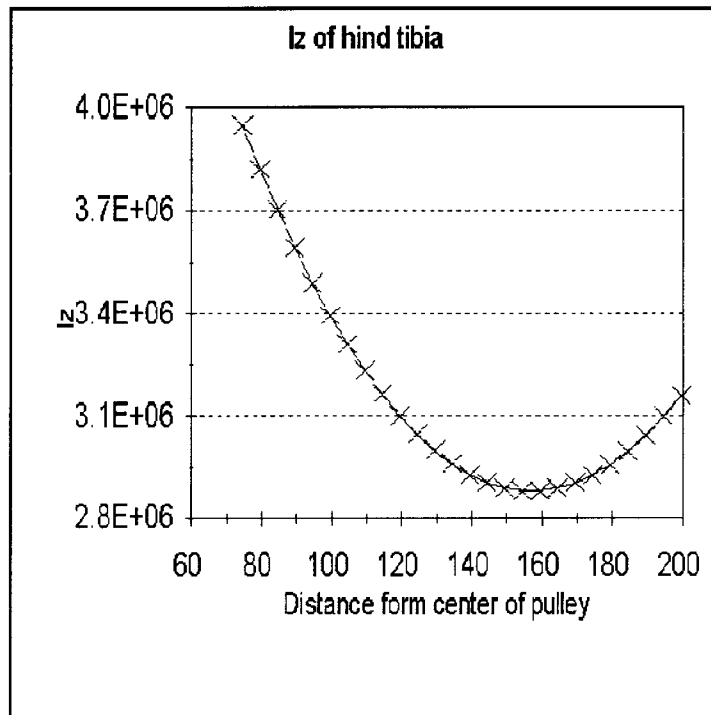


Figure A12 : Variation of I_z with distance from axis for hind tibia.

The moment of inertia shown in the above figures were calculated using equation (a1) through (a7), using a spreadsheet. The above figures shows the variation of moment of inertia about X and Z axis for femur and tibia for front, middle and hind legs. The Y component of moment of inertia are not shown, since they assumed to be constant. For calculating the moment of inertia there were some assumptions made, and some of the parts of the segments of the legs were completely neglected. From above figures the location of center of mass can be easily made, as the moment of inertia will be minimum at the center of mass of the segment of leg.